

BAR MODELLING PROGRESSION

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KEY MODELS

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INTRODUCTION

Bar modelling is used widely in Singapore, Japan and USA as an effective part of the Concrete, Pictorial, Abstract (CPA) approach to the mastery of mathematics. It allows children to see the relationship between the parts and the whole. Bars or boxes are used to represent known and unknown quantities.

Concrete materials are embedded alongside pictorial representations and abstract expressions to ensure procedural fluency and conceptual understanding are developed in tandem. It is not a method for problem solving but does reveal the mathematical structure beneath the problem and mathematical relationships between its component parts.

Many teachers have found the approach very effective in gaining an enduring understanding for children by providing many tangible and memorable experiences to draw upon in their learning.

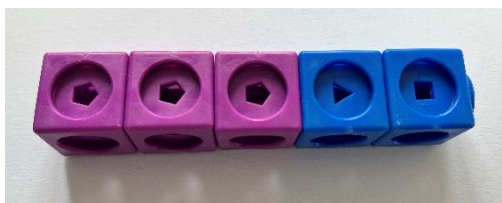
NATIONAL CURRICULUM

As we know, the aims of the national curriculum require the children to:

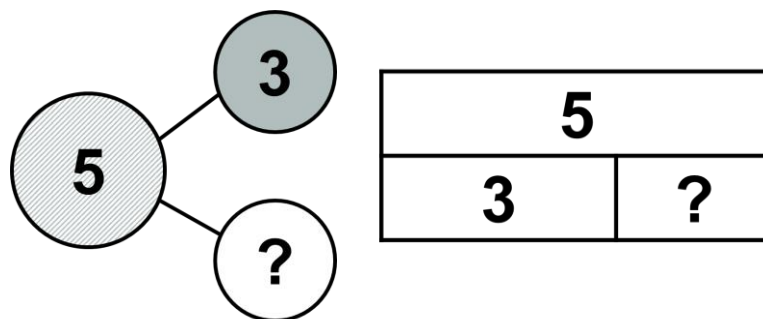
- become fluent ... through varied and frequent practice with increasingly complex problems [to] develop conceptual understanding
- reason mathematically
- solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication

For each year group and mathematical domain, relevant statements from the national curriculum programme of study are identified and exemplified using bar modelling as part of the CPA approach.

CONCRETE



PICTORIAL



ABSTRACT

I know that 3 and two more is equal to 5.

$$5 = 3 + 2$$

KEY MODELS

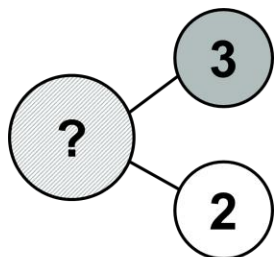
Addition and subtraction

Problems involving addition and subtraction have three possible unknowns. When given the value of two unknowns, the third can be found.

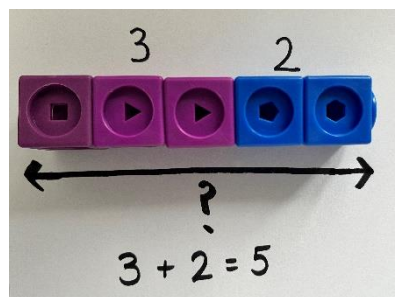
Addition models – total unknown

Aggregation (two quantities (parts) combined)

Three yellow sweets and two red sweets are on the table. How many sweets are on the table?

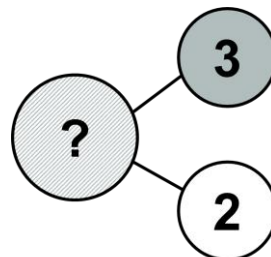


Three and two equals five.

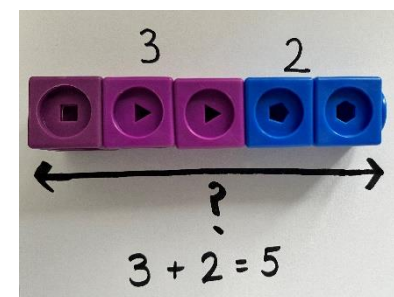


Augmentation (one quantity is increased by a provided amount)

There were three sweets on the table. Two more have been put on. How many sweets are on the table now?



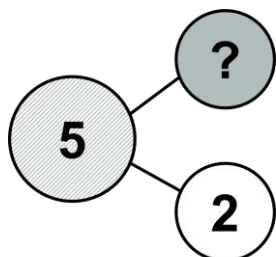
Two more than three is five.



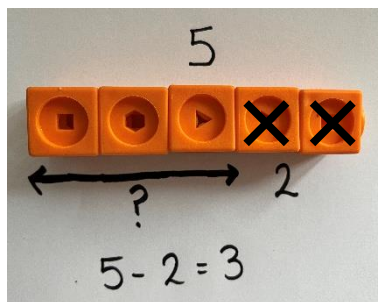
Subtraction models – one of the quantities (parts) is unknown

Take away (one quantity is decreased by a provided amount)

Five sweets were on the table. Tom ate two sweets. How many sweets are on the table now?

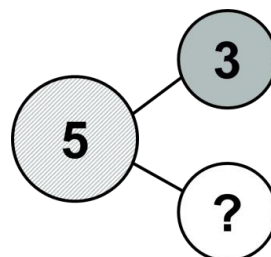


Two fewer than five is three.
Three is two fewer than five.

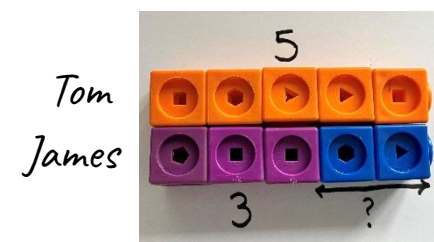


Comparison (finding the difference) *

Tom has five sweets and James has three sweets. How many more sweets does Tom have than James?



Five is two more than three.
Three is two fewer than five.



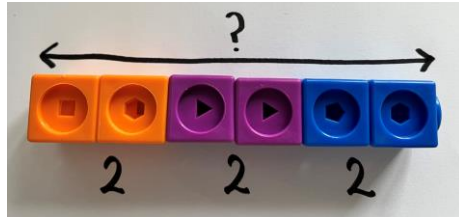
* Key model showing the relationship between addition and subtraction

Multiplication and division: EQUAL GROUPS

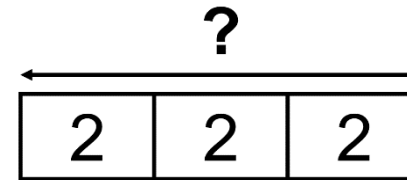
Problems involving multiplication and division have three possible unknowns – product, group size or number of groups. When given the value of two unknowns, the third can be found. In bar modelling, all blocks represent the replication of an equal unit.

Multiplication models – group size and number of groups provided but the product is unknown

There are three boxes with two teddy bears in each box. How many teddy bears are in total?



Two and two and two equals six.
Three groups with two in each group equals six.

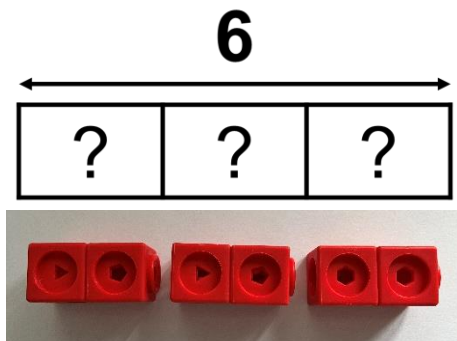


Three groups of two equals six.
Six is equal to three groups of two.

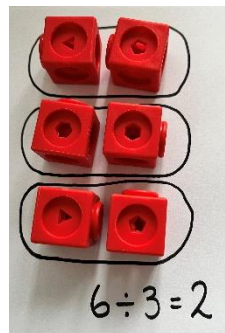
Division models – product is known but either the group size or number of groups is unknown

Sharing (product and number of groups is known but group size is unknown)

If six teddy bears are shared equally into 3 boxes, how many teddy bears will be in each box?

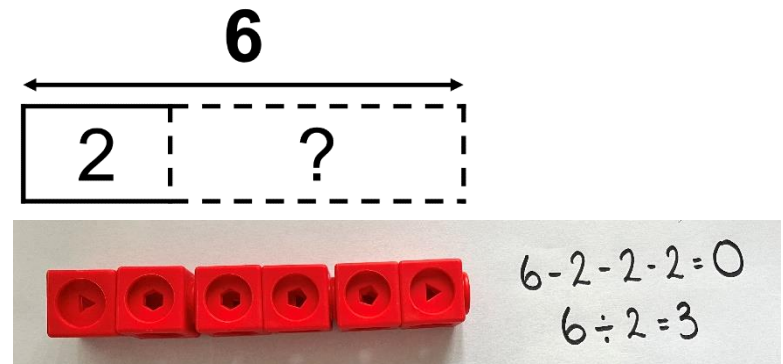


Six is equal to three groups with two in each group.



Grouping (product and group size is known but number of groups is unknown)

If six teddy bears are to be packed 2 to a box, how many boxes are needed?



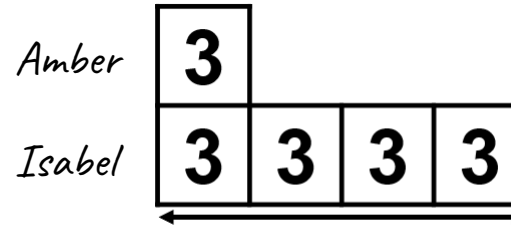
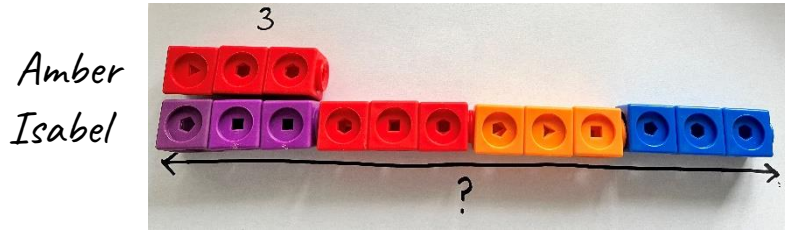
There are three twos in six.
Two can be taken from six, three times.
Three equal groups of two are equal to six.

Multiplication and division: COMPARISON

Problems involving multiplicative comparison occur when two or more sets are compared by showing one set is 'a number of times' larger or smaller than the other set(s). Bar modelling focuses on the multiplicative relationship between the sets.

Multiplication models – the smaller set and the multiplicative relationship is known but the product is unknown

Amber read three books. Isabel read four times as many books as Amber. How many books did Isabel read?

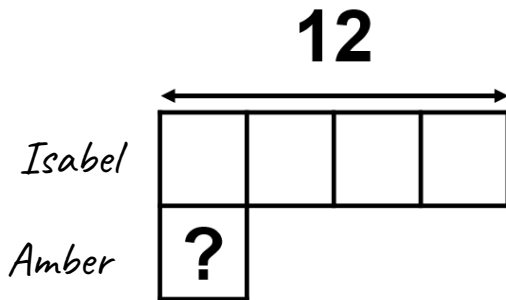


There are five equal groups altogether. Amber has one group and Isabel has four groups. All groups are equal. (ratio of 1:4) Amber has $\frac{1}{5}$ of all the books. (proportion)

Division models – the larger set is known but either the smaller set or the multiplicative relationship is unknown

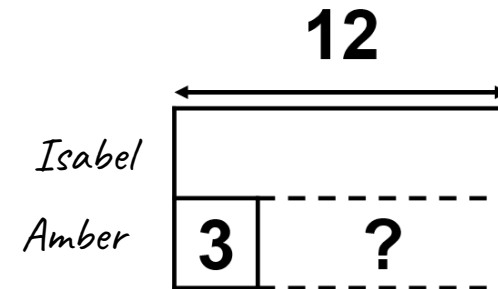
Sharing (the larger set and the multiplicative relationship is known but the smaller set is unknown)

Isabel read 12 books which is four times as many books as Amber read. How many books did Amber read?



Grouping (the larger set and the smaller set is known but the multiplicative relationship is unknown)

Isabel read 12 books and Amber read 3 books. How many times as many books did Isabel read as Amber did?



YEAR 1

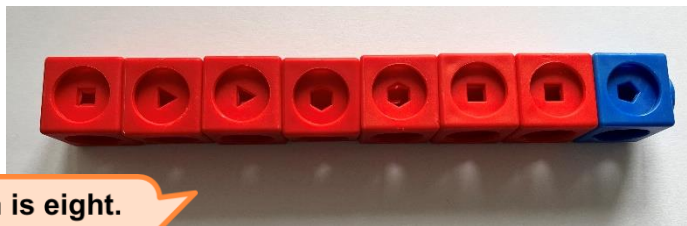
Number and place value

At this stage, children will be using multilink cubes and Cuisenaire Rods alongside their own pictorial representations (including a drawn bar model) which are then translated into abstract calculations and language.

- Count in multiples of twos, fives and tens



- Given a number, identifies one more and one less
- Identifies and represent numbers using objects and pictorial representations, including the number line and use the language of equal to, more than, less than (fewer), most, least



One more than seven is eight.
Eight is one more than seven.
Seven is one fewer than eight.



One fewer than seven is six.
Seven is one more than six.
Six is one fewer than seven.



There are more blue cubes than yellow cubes.
There are fewer yellow cubes than blue cubes.
They are not equal.

Addition and subtraction

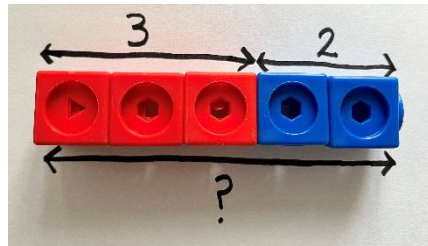
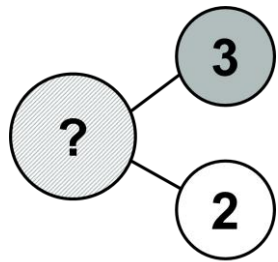
At this stage, children should be exposed to different models of addition and subtraction. It is important that they develop deep understanding of how addition and subtraction are related to each other – focus on language and how one model can be interpreted in many ways.

- Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs
- Represent and use number bonds and related subtraction facts within 20
- Add and subtract one-digit and two-digit numbers to 20, including zero
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$

Addition

Aggregation (two quantities combined)

Three yellow sweets and two red sweets are on the table. How many sweets are on the table?

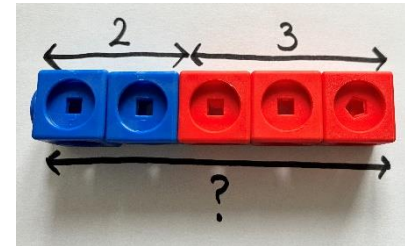


$$3 + 2 = 5$$

$$5 = 2 + 3$$

$$5 = 3 + 2$$

$$2 + 3 = 5$$

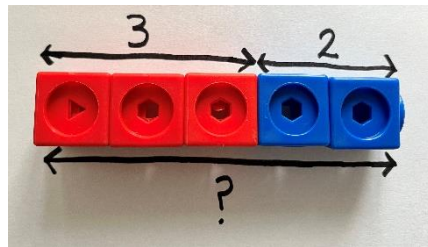
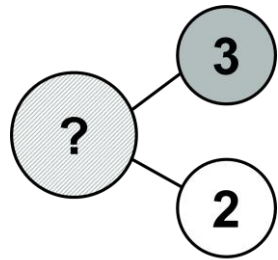


Three add two equals five.

Two add three equals five.

Augmentation (one quantity is increased)

There were three sweets on the table. Two more have been put on. How many sweets are on the table now?



$$3 + 2 = 5$$

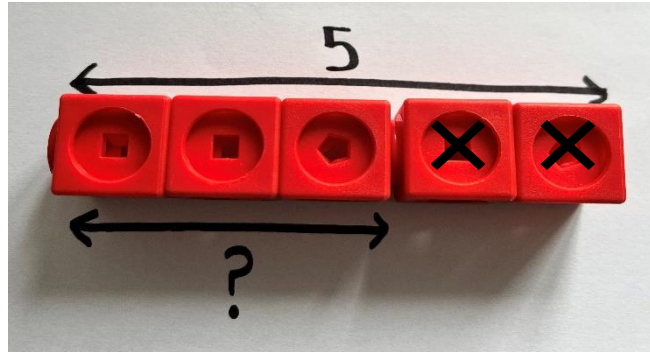
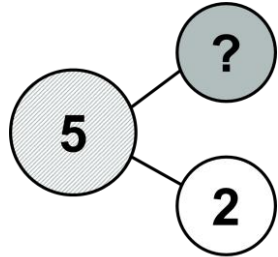
$$5 = 3 + 2$$

Three and two more makes five.

Subtraction

Take away (one quantity is decreased by a provided amount)

Five sweets were on the table. Tom ate two sweets. How many sweets are on the table now?



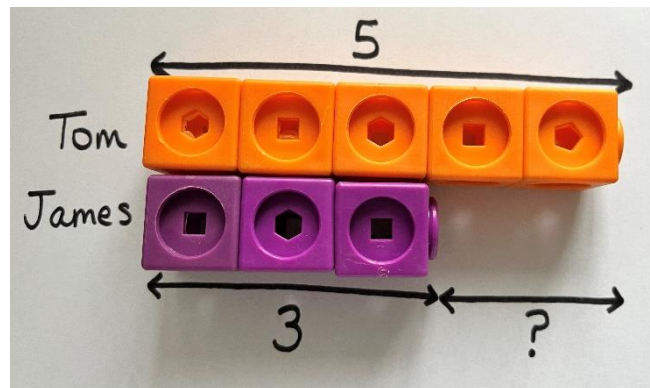
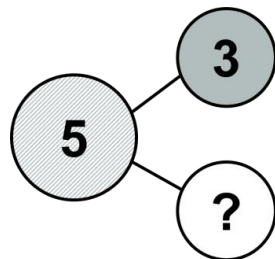
$$5 - 2 = 3$$

$$3 = 5 - 2$$

Two fewer than five is three.
Three is two fewer than five.

Comparison (finding the difference)

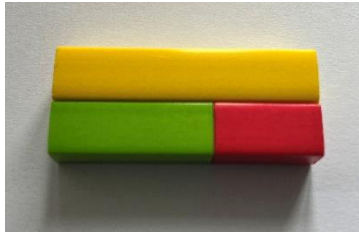
Tom has five sweets and James has three sweets. How many more sweets does Tom have than James?



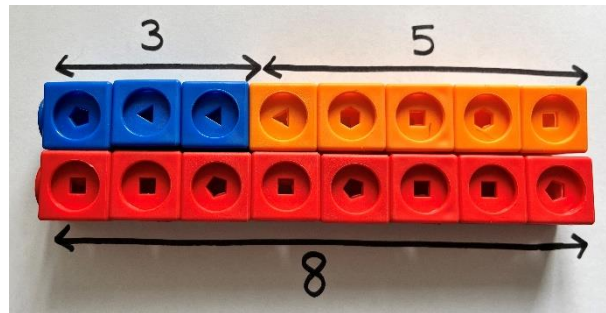
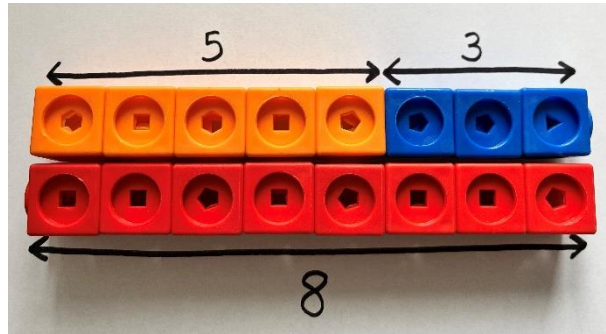
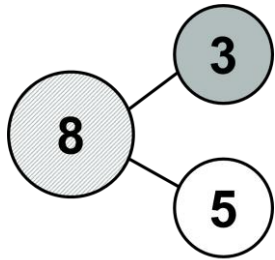
Five is two more than three.
Three is two fewer than five.

*Key model showing the relationship between addition and subtraction

Further examples of the comparison model for Year 1: exploring equality, more than, less/fewer than and discovering commutativity of addition. Cuisenaire rods can be used to explore the generalised relationship of addition and subtraction.



Yellow is green add red.
 Red add green equals yellow.
 Yellow take away green equals red.
 Yellow take away red equals green.



$$5 + 3 = 8$$

$$3 + 5 = 8$$

$$8 = 5 + 3$$

$$8 = 3 + 5$$

Three more than five is eight.
 Three add five is equal to eight.
 Five add three is equal to eight.
 Five more than three is 8.

Eight subtract five is three.
 Five fewer than eight is three.
 Eight subtract three is five.
 Three fewer than eight is five.

Ensure that children explore the models and can move the position of the unknown quantity.

- How would I find the number of blue cubes?
- Can you give me a story problem which makes the five the unknown?
- Can you tell me about the model using 'greater than'?
- What else can you tell me?

$$8 - 5 = 3$$

$$8 - 3 = 5$$

$$5 = 8 - 3$$

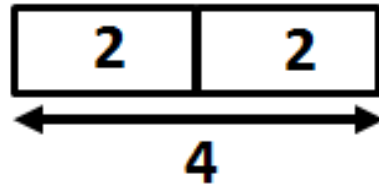
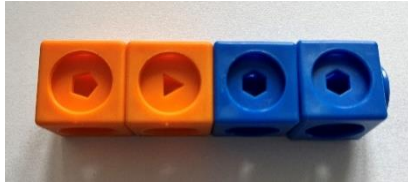
$$3 = 8 - 5$$

Multiplication and division

At this stage, it is important that the children begin to understand that multiplication and division involve the replication of a single unit a number of times and start to explore unitisation e.g., one block is worth two.

- Solve one-step problems involving multiplication and division, by calculating the answer by using concrete objects, pictorial representations and arrays with the support of the teacher

Doubling and halving



Two and two makes four.
Double two is equal to four.
Half of four is two.

$$2 + 2 = 4$$

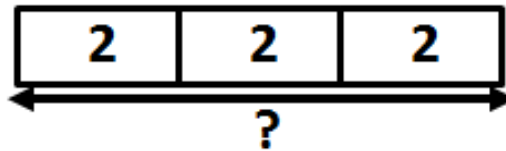
$$2 \times 2 = 4$$

$$4 - 2 - 2 = 0$$

$$4 \div 2 = 2$$

Multiplication (equal groups)

Amber wants to give her three friends two lollies each. How many lollies does she need?



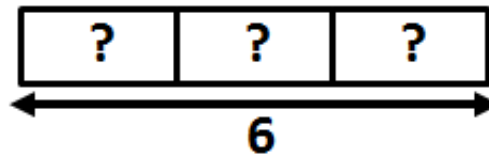
Two and two and two makes six.
Three groups of two make six.

$$2 + 2 + 2 = 6$$

$$3 \times 2 = 6$$

Division (sharing)

Amber has six lollies. She wants to share them equally between her three friends. How many lollies does each of her friends get?



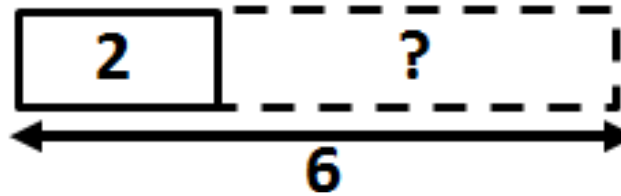
Six shared into three equal groups gives you two in each group.

$$6 \div 3 = 2$$

Division (grouping)

Amber has six lollies. She wants to give each of her friends two lollies. How many friends can she give lollies to?

$$6 \div 2 = 3$$

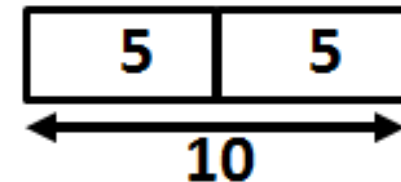


Six grouped into twos gives you three groups.

Fractions

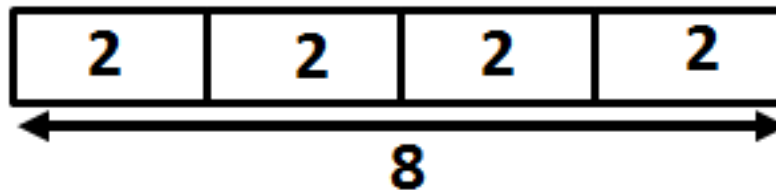
- Recognise, find and name a half as one of two equal parts of an object, shape or quantity
- Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity

Cuisenaire Rods can be used to explore generalisation in fractions.



The orange is double the yellow.
The yellow is half of the orange.
Yellow add yellow equals orange.
Orange take away yellow equals yellow.

Four reds are equal to a brown.
A quarter of brown is one red.
Red add red add red add red equals brown.



YEAR 2

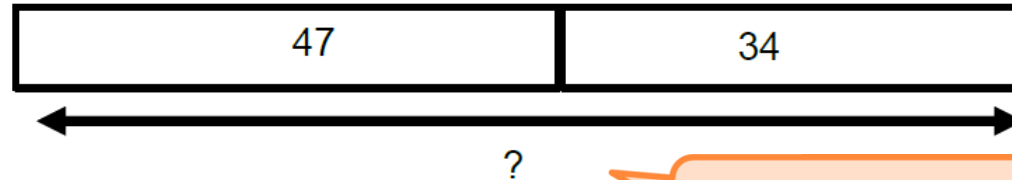
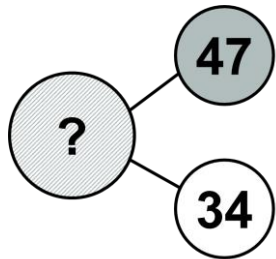
Addition and subtraction

- Solve problems with addition and subtraction:
 - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
 - applying their increasing knowledge of mental and written methods
- Recall and use addition and subtraction facts up to 20 fluently, and derive and use related facts up to 100
- Add and subtract numbers using concrete objects, pictorial representations and mentally
- Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

Addition

Addition (aggregation)

Amber has 47 pence and Isabel has 34 pence. How much do they have altogether?



**34 and 47 equal 81 altogether.
The total of 47 and 34 is 81.**

$$47 + 34 = 81$$

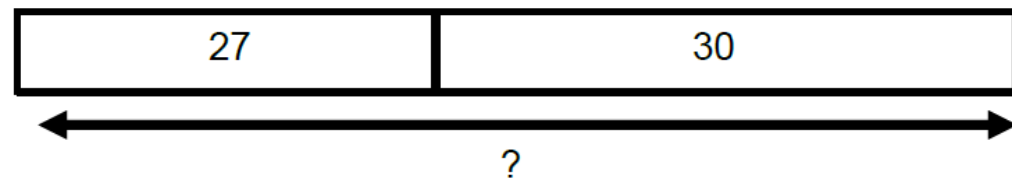
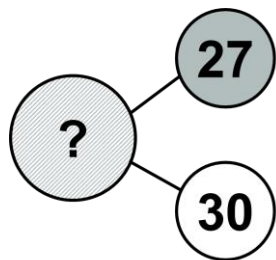
$$34 + 47 = 81$$

$$81 = 34 + 47$$

$$81 = 47 + 34$$

Addition (augmentation)

Amber has 27 gel pens. She buys 30 more. How many gel pens does she now have?



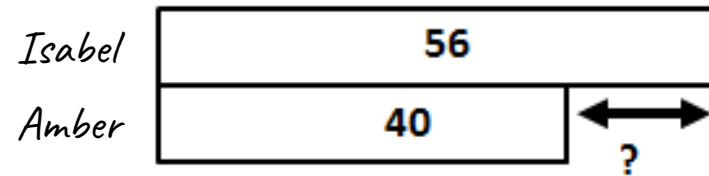
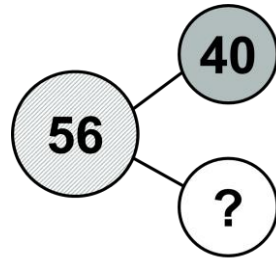
$$27 + 30 = 57$$

$$57 = 27 + 30$$

Subtraction

Subtraction (comparison)

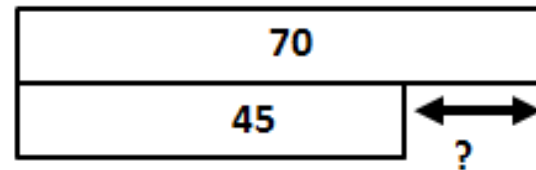
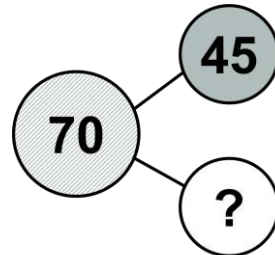
Amber has 40 buttons. Isabel has 56 buttons. How many fewer buttons does Amber have than Isabel?



40 buttons is 16 fewer than 56 buttons.

Further examples:

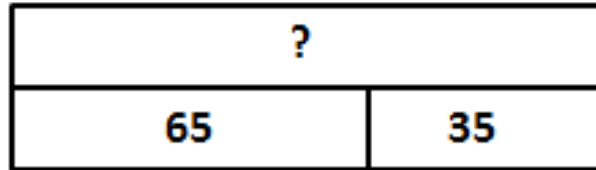
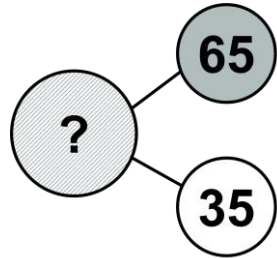
Amber needs 70g of flour for her cake. There is 45g of flour left in the bag. How much more flour does she need?



45g is 25g less than 70g.
70g is 25g more than 45g.

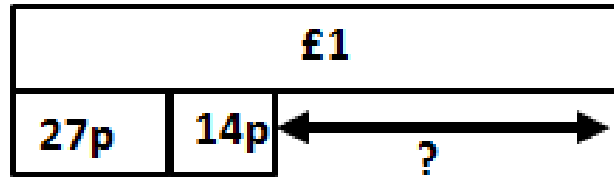
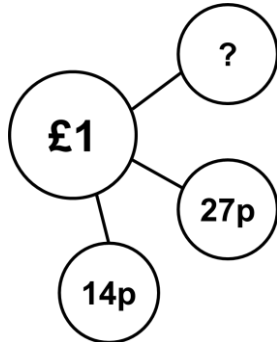
Further examples of addition and subtraction models for Year 2: exploring the structure of mathematical problems, with a focus on the meaning of 'more than' and 'less than / fewer than' in context.

Tom has 65cm of ribbon. This is 35cm less than Sam has. How much ribbon does Sam have?



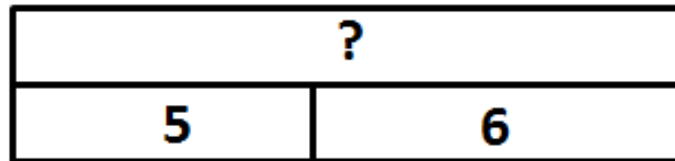
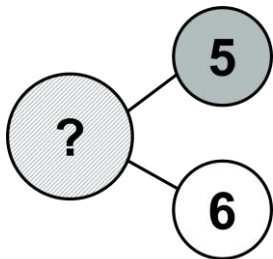
Sam has 100cm of ribbon.

Ellie has £1 pocket money. She spent 27p on a pen and 14p on a rubber. How much money does she have left?



Ellie has 59p left.

I think of a number then subtract 6. The answer is 5. What was my number?



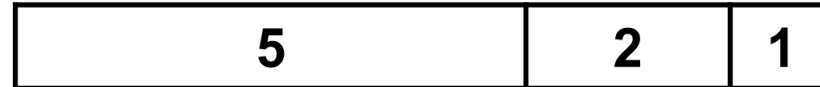
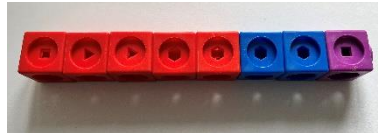
The number was 11.

Addition and subtraction

- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems

Exploring commutativity and associative law

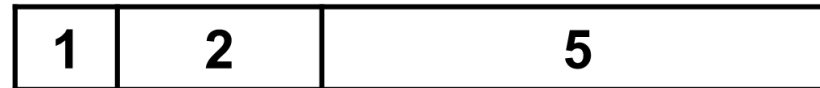
$5 + 2 + 1$



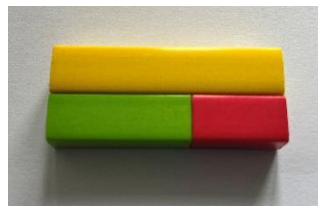
$1 + 5 + 2$



$1 + 2 + 5$



Cuisenaire Rods can be used to explore the generalised relationship between addition and subtraction.



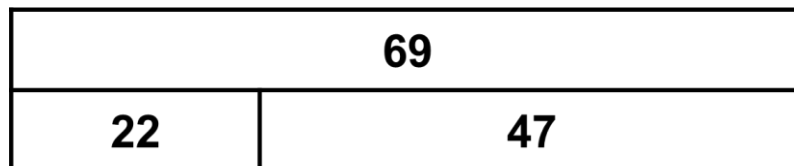
Does red equal green subtract yellow?

Yellow is green add red.
Red add green equals yellow.
Yellow subtract green equals red.
Yellow subtract red equals green.

Sally writes an answer to the calculation below.

$69 - 47 = 22$

What calculations could she do to check her answer?



$22 + 47 = 69$

$47 + 22 = 69$

$47 = 69 - 22$

$22 = 69 - ?$

Multiplication and division

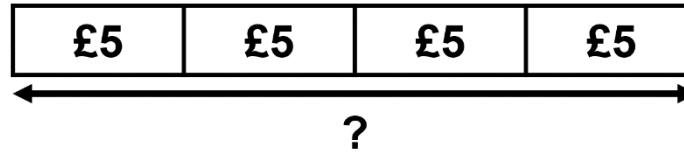
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x) and division (÷) and equals (=) signs
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in contexts

Multiplication (equal groups)

James buys four tickets for the football match. Each ticket costs £5. How much does he spend?

$$£5 + £5 + £5 + £5 = £20$$

$$4 \times £5 = £20$$



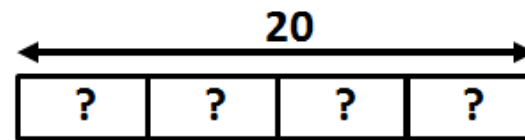
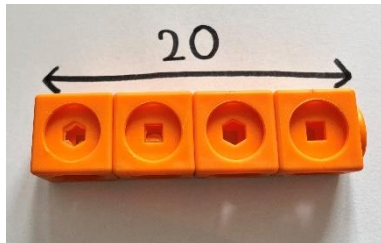
Four groups of £5 is £20.
£5 four times is £20.

It is important that children start to understand unitisation at this point.

Division (sharing)

There are 20 children in the hall. The teacher wants to put the children into 4 equal teams. How many children will be in each team?

$$20 \div 4 = 5$$

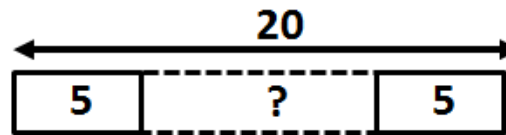
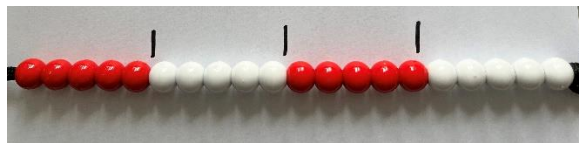


20 shared equally between 4 groups results in 5 in each group.

Division (grouping)

Claire wants to put 5 biscuits on each plate. She has 20 biscuits. How many plates does she need?

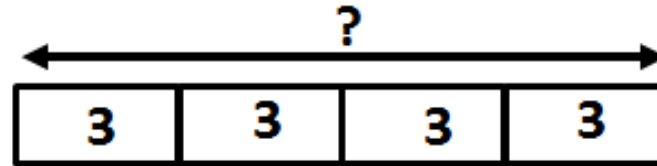
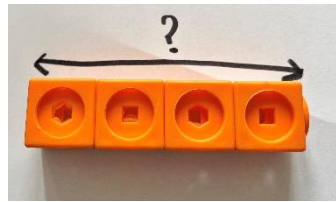
$$20 \div 5 = 4$$



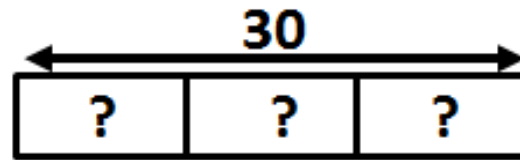
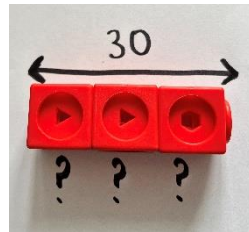
There are 4 groups of 5 in 20.

Further examples of multiplication and division models for Year 2:

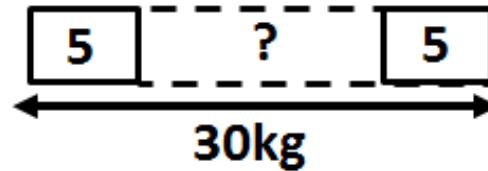
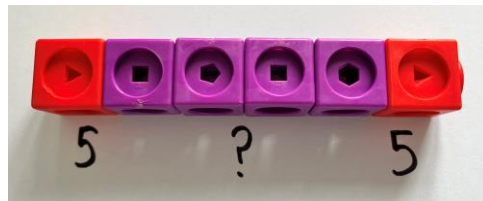
Claire has 4 building blocks. Each building block is 3m long. If she lays them end to end to build a wall, how long is the wall?



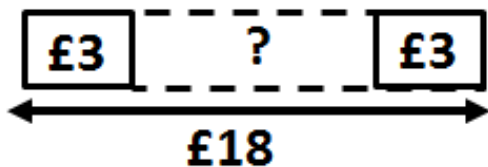
Amber has a ribbon 30cm long. She cuts it into 3 equal pieces. How long is each piece?



Isabel bought some bags of apples. Each bag has a mass of 5kg. The mass of all the bags is 30kg. How many bags of apples did Isabel buy?



Amber bought some books at the school fair for £18. Each book cost £3. How many books did she buy?



Fractions

- Recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity

Cuisenaire Rods and multilink cubes can be used to explore different fractions.

Half

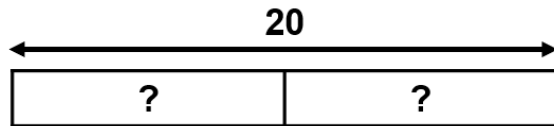


What fractions do you see?

If the orange rod is worth 6, what is the value of a yellow rod?

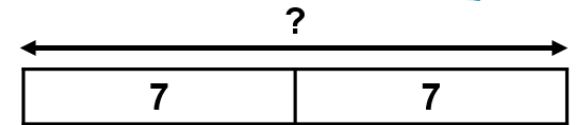
If each of the yellow rods is worth 8, what is the value of the orange rod?

If the orange rod is worth 100, what is the value of a yellow rod?

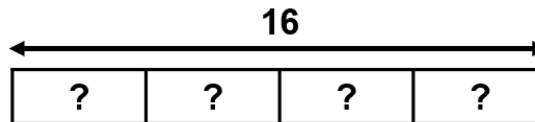


What is half of 20?

If half of a number is 7, what is the number?

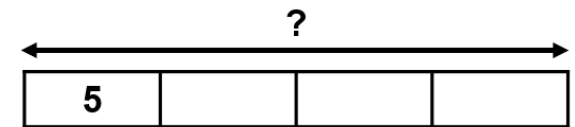


Quarters



What is one quarter of 16?

What is $\frac{3}{4}$ of 16?

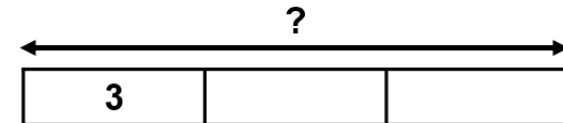
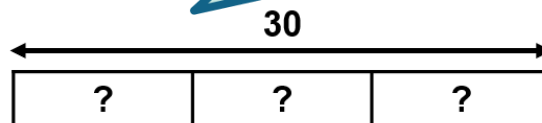
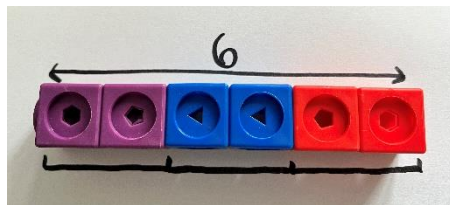


Thirds



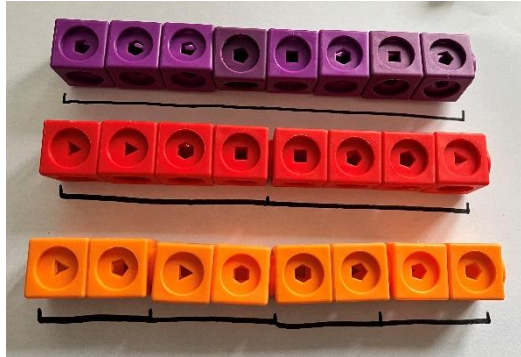
If 30 is the whole, what is $\frac{1}{3}$?

If one quarter of a number is 5, what is the number?



Fractions

- Write simple fractions, for example, $\frac{1}{2}$ of 6 = 3, and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$



12			
6		6	
3	3	3	3



Which other colour rods can you use to show the whole and both halves and quarters?

- Pupils should count in fractions up to 10 (non-statutory)



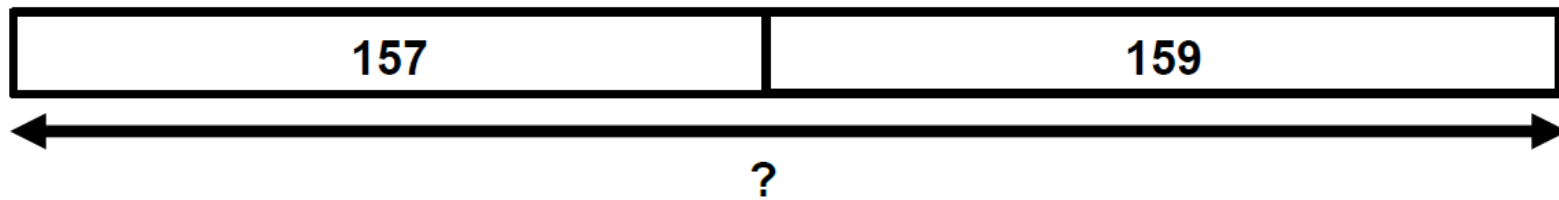
YEAR 3

Addition and subtraction

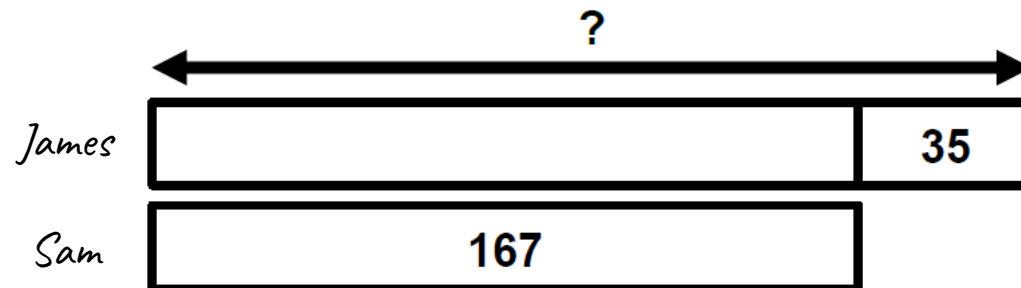
- Add and subtract numbers mentally, including:
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- Estimate the answer to a calculation and use inverse operations to check answers
- Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction

Addition

In a cricket match, James' team score 157 runs in the first innings and 159 in the second innings. How many runs did they score in total?

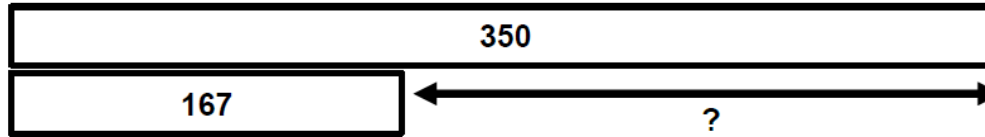


James scored 35 more points than Sam. Sam scored 167 points in his game. How many points did James score?

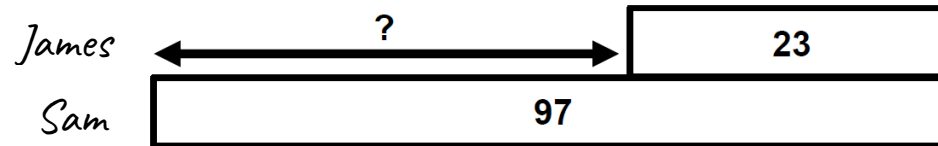


Subtraction

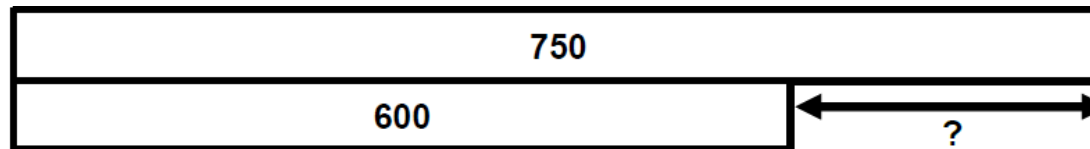
There are 350 pages in Amber's book. On Tuesday, she read 167 pages of her book. On Wednesday, she read the rest of the book. How many pages did she read on Wednesday?



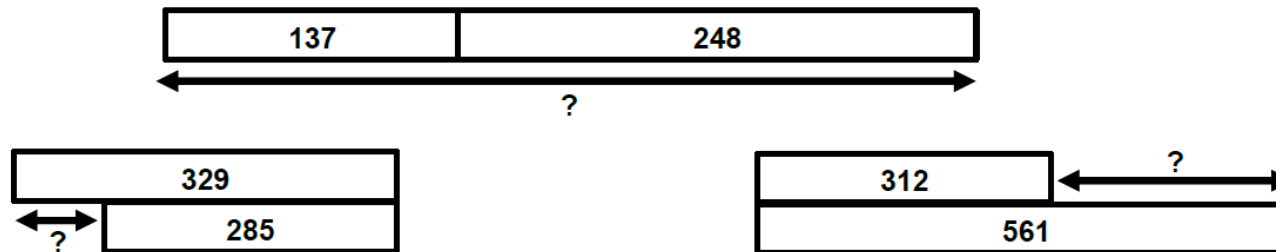
James has collected 23 fewer football stickers than Sam. Sam has collected 97. How many football stickers has James collected?



A bag of flour weighed $\frac{3}{4}$ kg. Nicola used 600g of flour. How much flour is left in the bag?



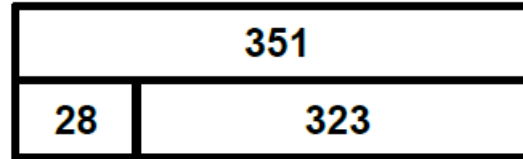
What story problems could these bar models represent?



Check answers using the inverse

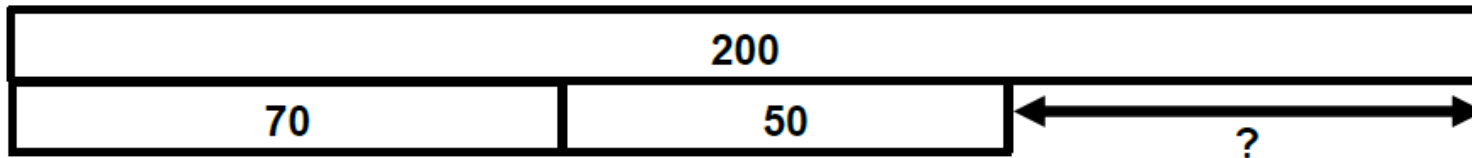
For the calculation $323 + 28$, Amber undertakes the calculation $351 - 323$.

Can you explain why she chose to do this?



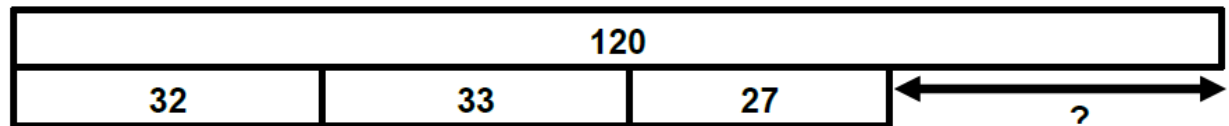
Multi-step problems

Gill has £2 pocket money. She spends 70p on a comic and 50p on a bar of chocolate. How much change should she get?



Complete this table:

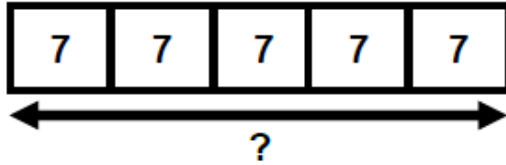
Year group	Number of children
3	32
4	33
5	?
6	27
Total	120



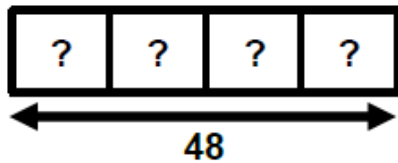
Multiplication and division

- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

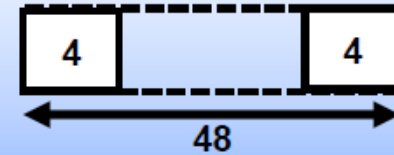
$5 \times 7 = ?$



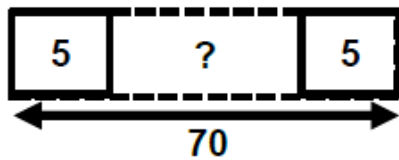
$4 \times ? = 48$



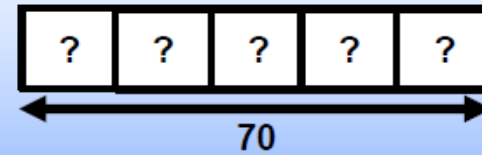
Or (due to commutativity)



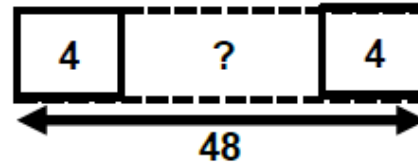
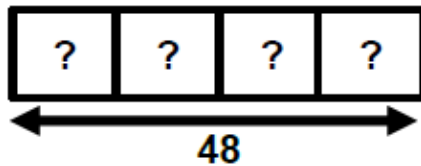
$70 \div 5 = ?$



Or (due to commutativity)



$48 \div ? = 4$

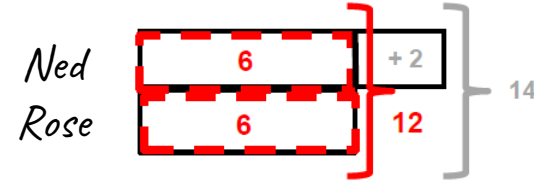
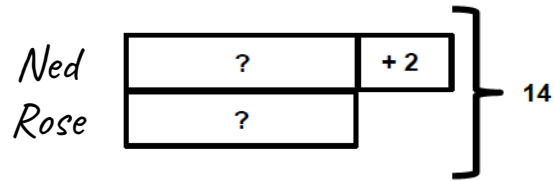


It could either be the number of groups or the size of each group that is unknown.

Multiplication and division

- Solve problems, including missing number problems, involving multiplication and division

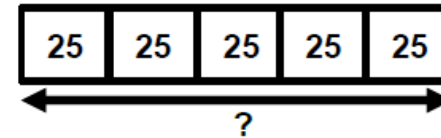
Ned and Rose have some biscuits. Altogether, they have 14 biscuits. Ned has 2 more biscuits than Rose. How many biscuits do they each have?



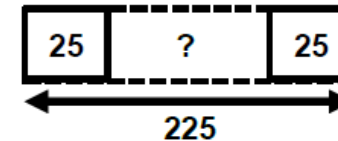
Measures

One length of the swimming pool is 25 metres.

a) Rachel swims five lengths of the pool. How far does Rachel swim altogether?

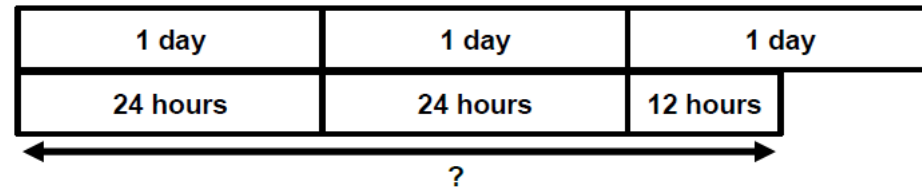


b) Sam swims 225m in the pool. How many lengths does he swim?



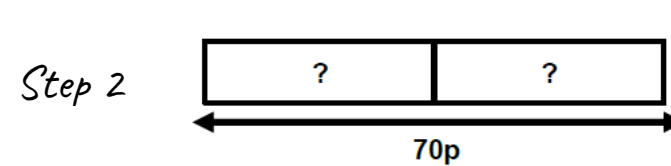
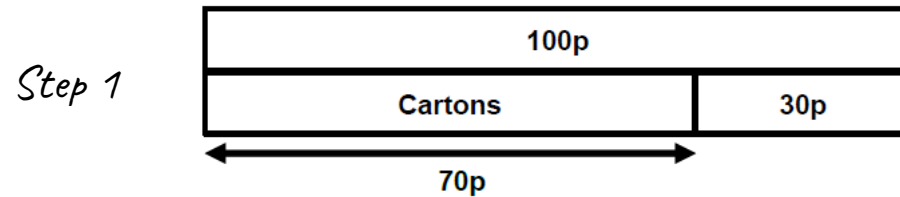
Time

How many hours are there in two and a half days?



Money

I had one pound. I bought two cartons of drink and received 30p change. How much did each carton cost?



Multiplication and division

- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

At this stage, focus upon the multiplicative relationship between the bars.



What do you notice?

... is twice as many as ...

... is three times as many as ...

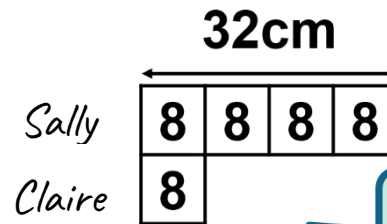
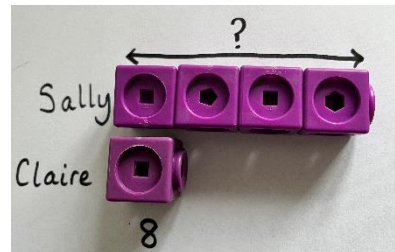
... is three times smaller than ...



Can you change this model to show that C is four times as big as D?

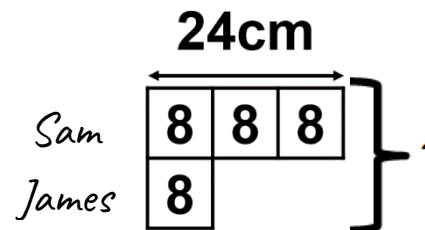
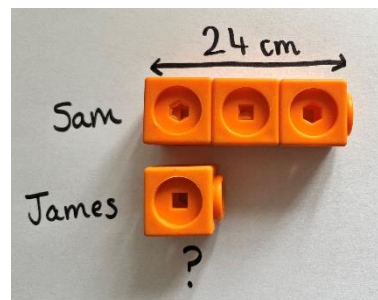
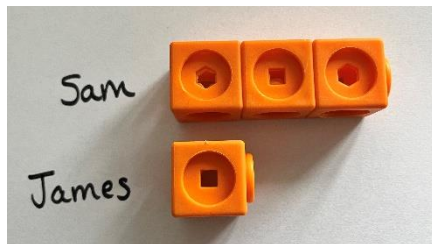
YEAR 3

Claire builds a tower that is 8 cubes tall. Sally builds a tower that is 4 times as tall. How tall is Sally's tower?



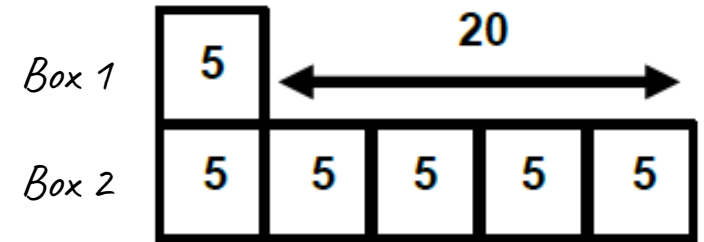
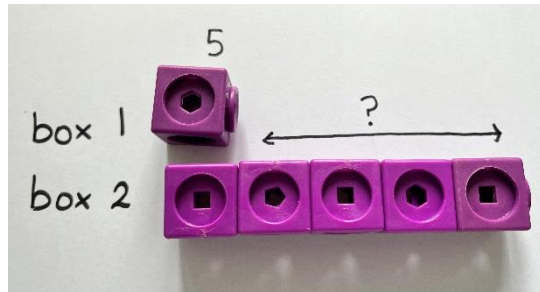
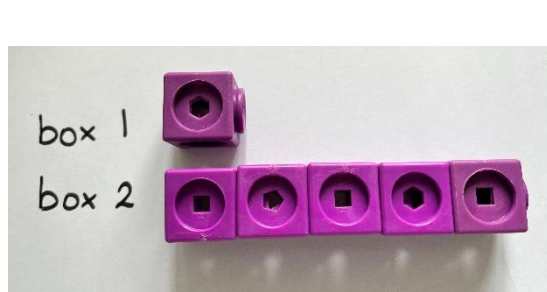
How much smaller is Claire's tower than Sally's tower?

Sam's ribbon is three times as long as James's ribbon. Sam's ribbon is 24cm long. How long is James's ribbon?

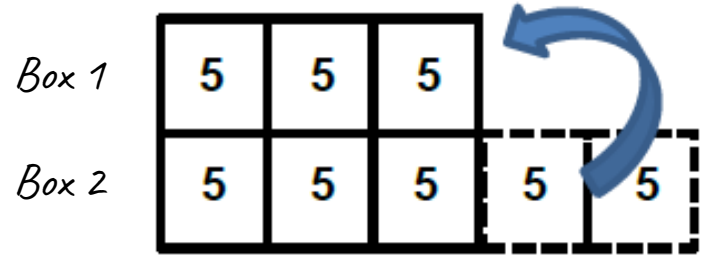
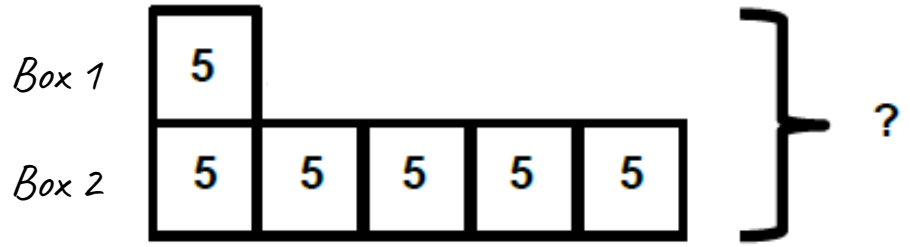


What is the total length of the ribbons?

Leigh is helping in the school library. She is packing books into two boxes. The first box has 5 books in it. The second box has five times as many books in it as the first box. How many more books does the second box hold than the first?



How many books in total have been packed into the two boxes?

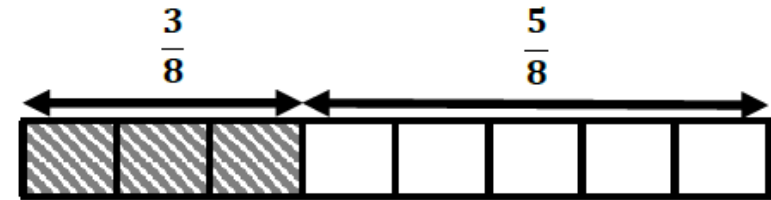
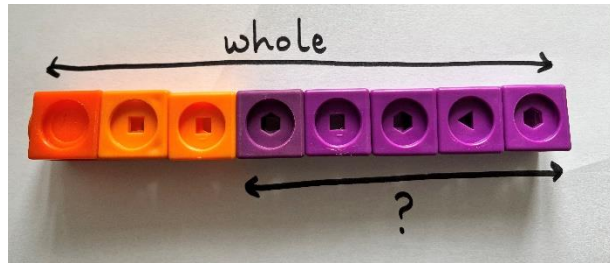


How many books need to be taken out of box 2 and placed in box 1 so that there is an equal number of books in each box?

Fractions

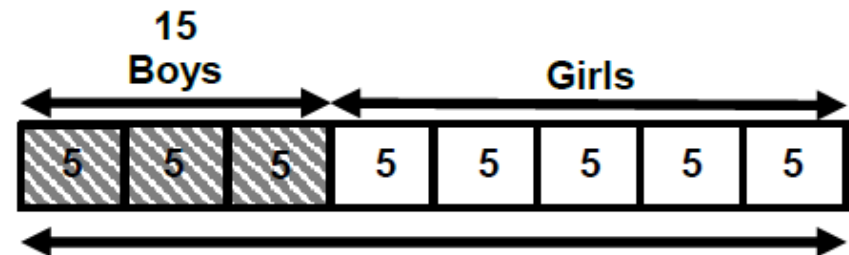
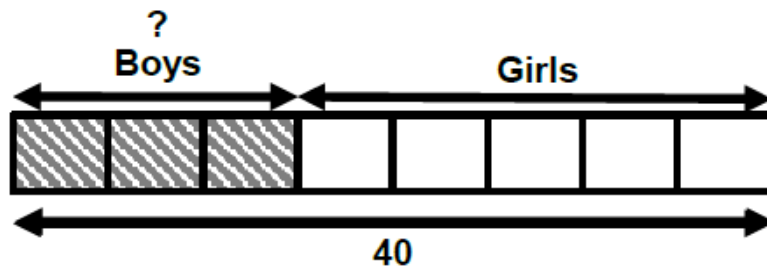
- Recognise, find and write fractions of discrete set of objects; unit fractions and non-unit fractions with small denominators

$\frac{3}{8}$ of a class are boys. What fraction of the class are girls?



YEAR 3

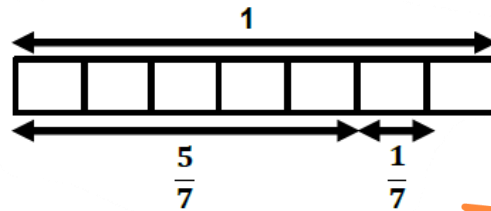
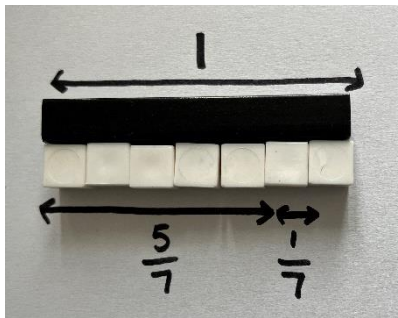
If there are 40 children in the class, how many boys are there?



Fractions

- Add and subtract fractions within the same denominator within one whole

$$\frac{5}{7} + \frac{1}{7} = ?$$



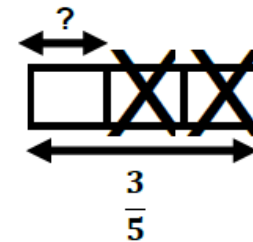
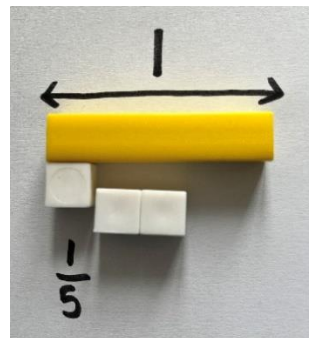
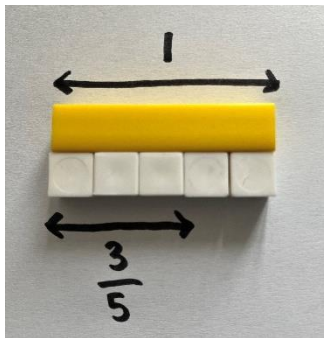
Ensure that children can define the fractions with reference to the whole.

$\frac{5}{7}$ add $\frac{1}{7}$ is equal to $\frac{6}{7}$ of the whole.

Subtraction with fractions

Take away model

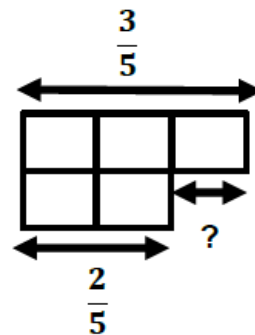
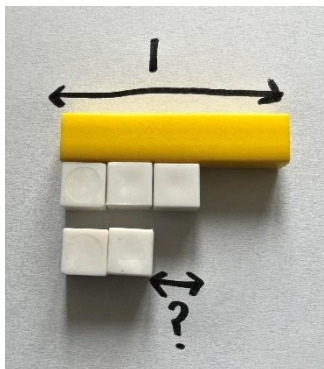
$$\frac{3}{5} - \frac{2}{5} = ?$$



$\frac{3}{5}$ take away $\frac{2}{5}$ is equal to $\frac{1}{5}$ of the whole.

Comparison model

$$\frac{3}{5} - \frac{2}{5} = ?$$



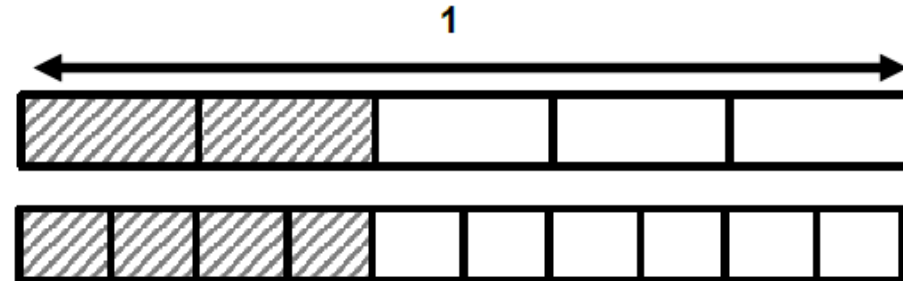
The difference between $\frac{3}{5}$ and $\frac{2}{5}$ is $\frac{1}{5}$ of the whole.

Fractions

- Recognise and show, using diagrams, equivalent fractions with small denominators

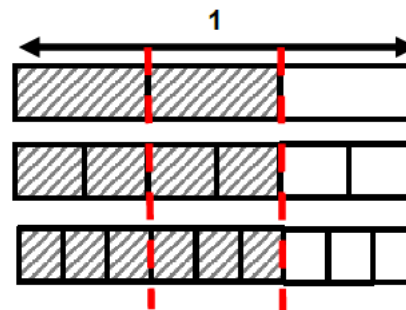
Children could explore this with Cuisenaire Rods first and then with bar models.

Find equivalent fractions to $\frac{2}{5}$.



$$\frac{2}{5} = \frac{4}{10}$$

Find equivalent fractions to $\frac{2}{3}$.

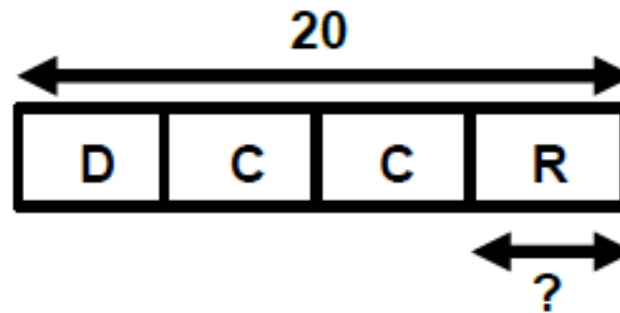


$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$$

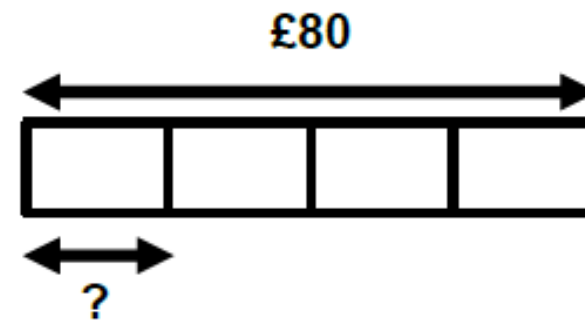
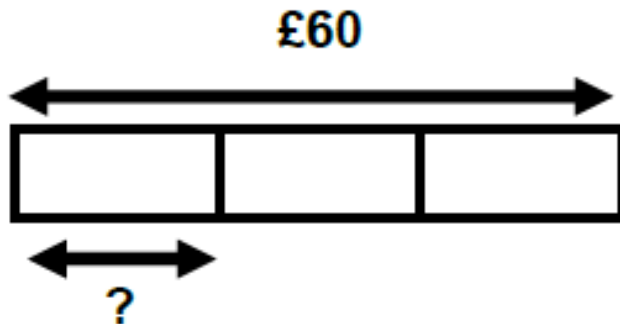
Fractions

- Solve problems that involve all of the above

Sally has 20 stickers on her page. One quarter of them are dog stickers. One half of them are cat stickers. The rest are rabbit stickers. How many rabbit stickers are on the page?



What is the larger amount; one third of £60 or one quarter of £80?

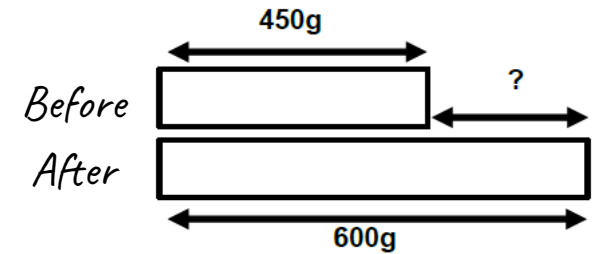
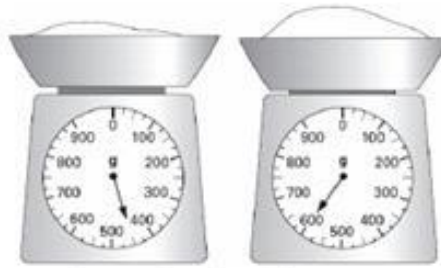


YEAR 4

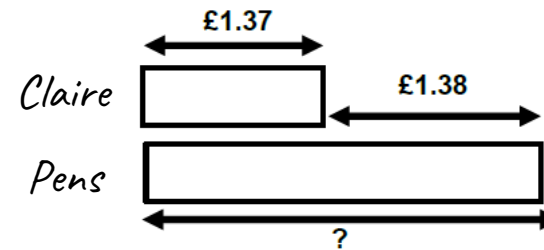
Addition and subtraction

- Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

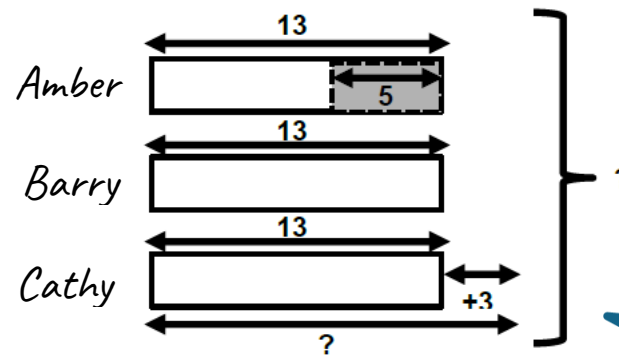
Emily is making a cake. She puts flour on the scales. Then she adds the sugar. How much sugar does she add?



Claire has £1.37. She needs £1.38 more to buy a packet of pens. How much do the pens cost?



Amber, Barry and Cathy run a 50m race. Barry's time is 13 seconds. Amber finishes 5 seconds before Barry. Cathy finishes 3 seconds after Barry. What is Cathy's time?



What is the total time taken by all three children?

Multiplication and division

- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers

5×0



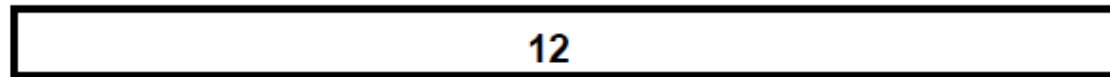
number of groups x size of each group = total number

Explore concrete ideas such as 5 plates, each with no biscuits on.
Consider the total number of biscuits.

Division

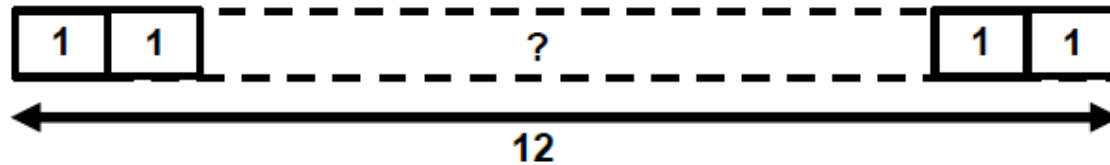
Division (sharing)

$12 \div 1$



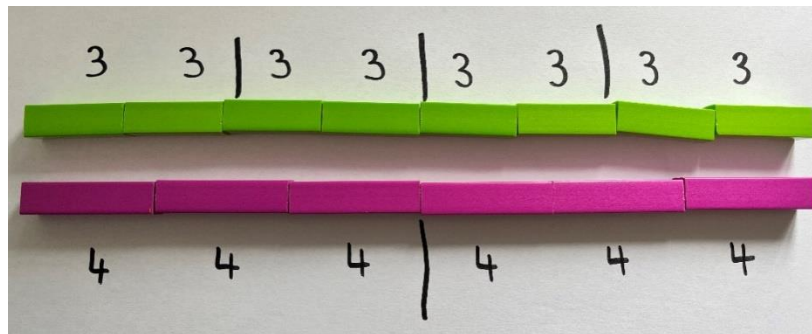
Division (grouping)

$12 \div 1$



Associative law – multiplying together three numbers

$(2 \times 3) \times 4 = 2 \times (3 \times 4)$

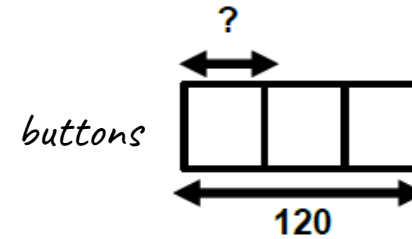
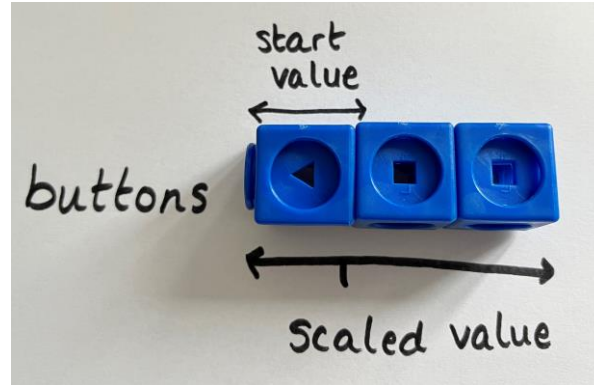


Multiplication and division

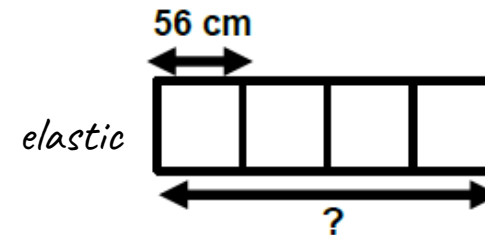
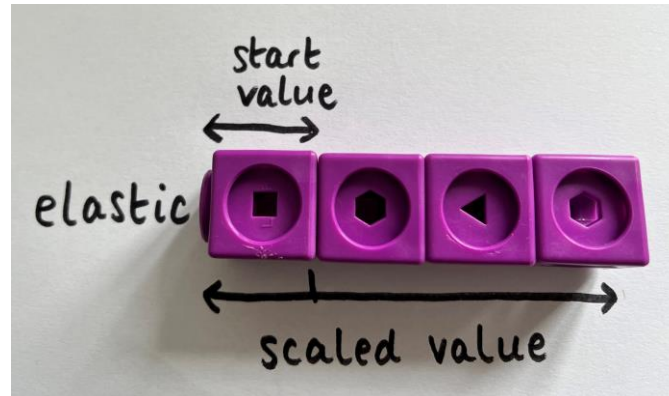
- Solve problems involving multiplication and adding, including using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Integer scaling

Louise collects buttons. She bought some new buttons to add to her collection. She now has 120 buttons, which is three times as many buttons as she started with. How many buttons did she start with?



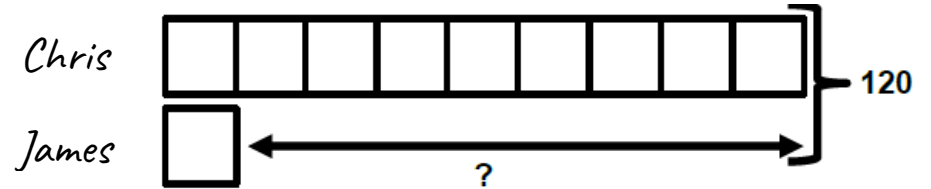
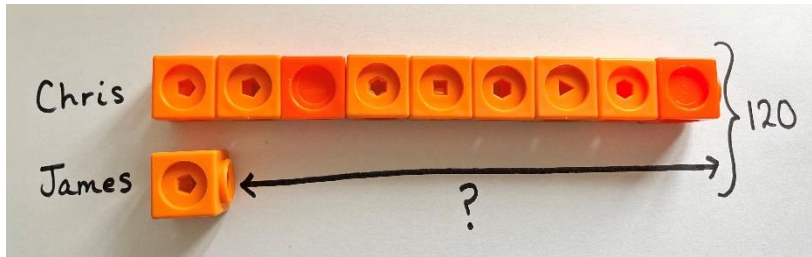
A length of elastic is stretched four times longer than its original size. It was 56cm long. How long is it now?



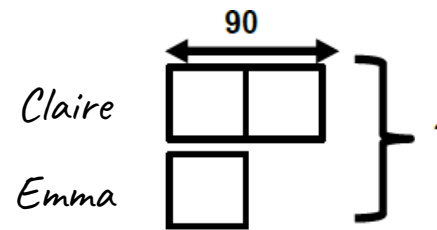
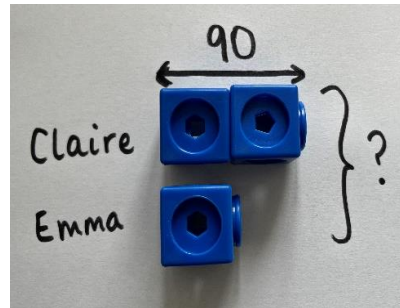
How much longer is it now?

Correspondence problems

Chris has 9 times as many Lego figures as James. Together they have 120 Lego figures. How many more Lego figures does Chris have than James?

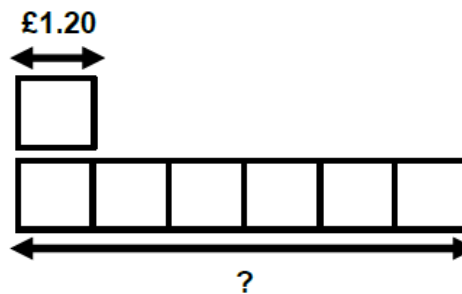
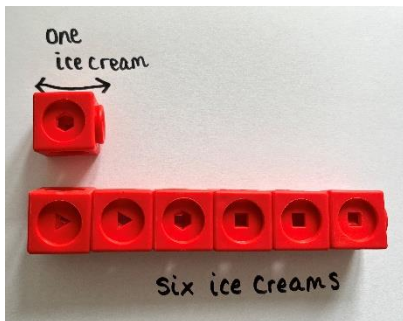


Claire has twice as many stickers as her friend Emma. Claire has 90 stickers. How many stickers do they have together?



Rate (non-statutory until Year 5)

An ice cream costs £1.20. Louise wants to buy 6 ice creams. How much will the ice creams cost?

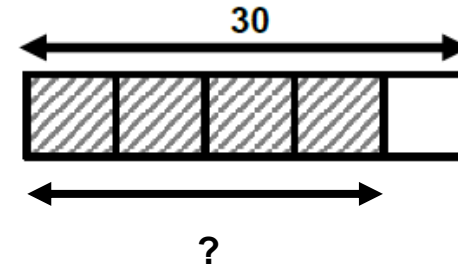
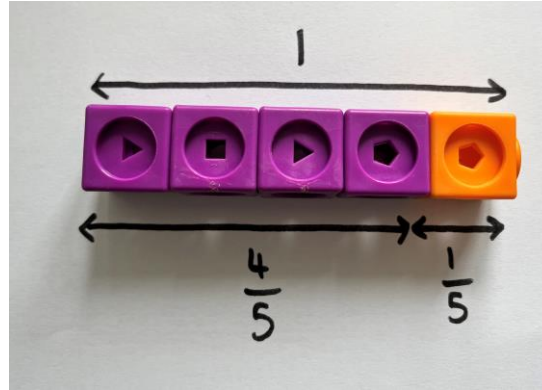


If Louise pays with a £10 note, how much change will she receive?

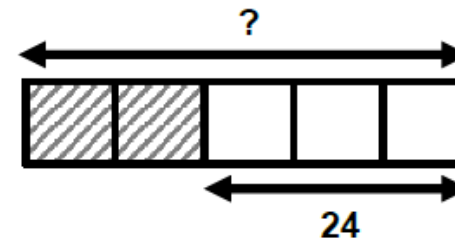
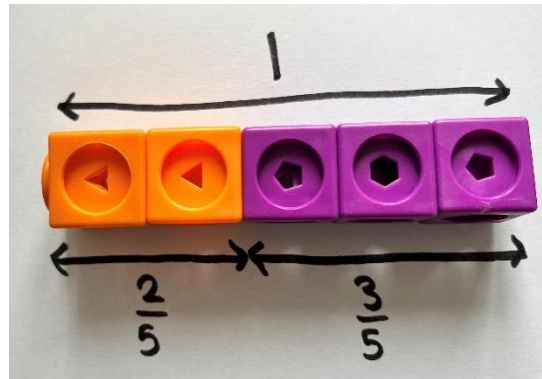
Fractions

- Recognise and show, using diagrams, families of common equivalent fractions ([click to see Year 3 progression](#))
- Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number (see models below)
- Add and subtract fractions with the same denominator ([click to see Year 3 progression](#))

Sally buys four fifths of the shop's apples. If the shop had 30 apples, how many apples did she buy?



James had some football cards. He gave two fifths away. He now has 24 cards. How many did he have to start with?



YEAR 5

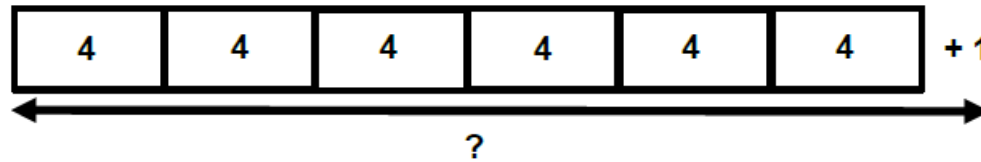
Multiplication and division

- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and **interpret remainders appropriately for the context**

The aim of these examples is to support understanding the bar model with remainders

Multiplication (equal groups)

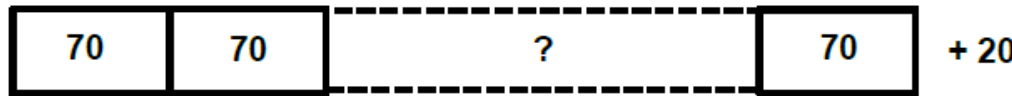
Tracey put 4 seeds into each of her pots. She uses 6 pots and has 1 seed left over. How many seeds did she start with?



Why is the 'one' not in a block?

Division (grouping)

Carl has 580 sweets. He wants to put 70 sweets in each bag. How many bags can he fill?

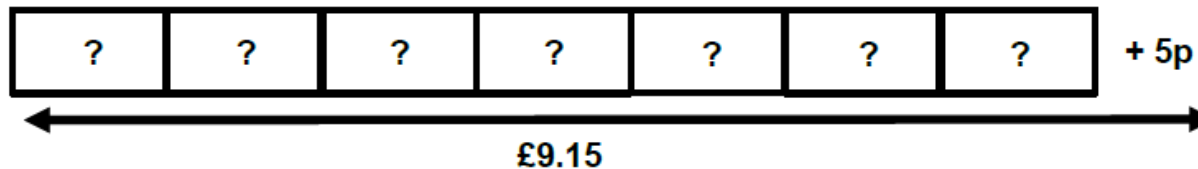


What does this number mean?

These are the sweets that are left over after filling the bags.

Division (sharing)

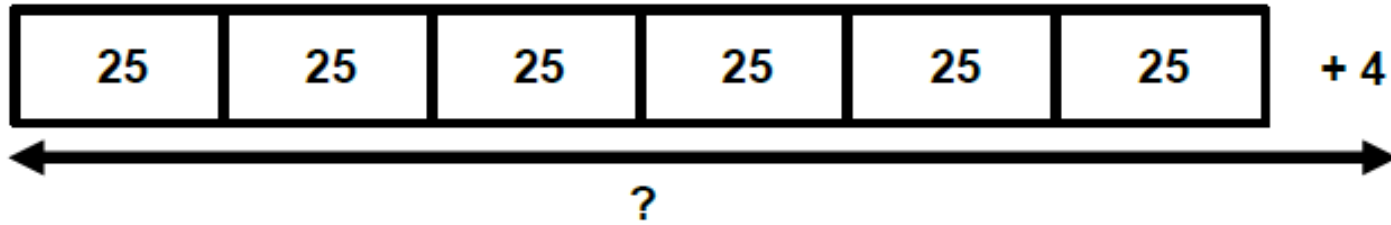
Grandad has been collecting his change in a jar. He has £9.15. He wants to share the money equally between seven grandchildren. How much money will each child receive?



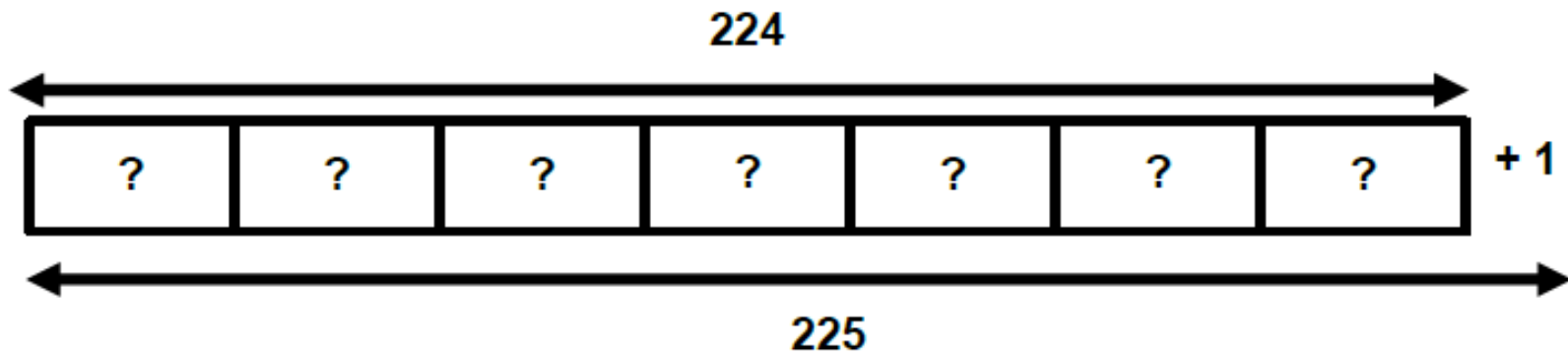
Why does the line need to include the remainder?

Exploring division notation to gain deeper understanding

$$? \div 6 = 25 \text{ r } 4$$



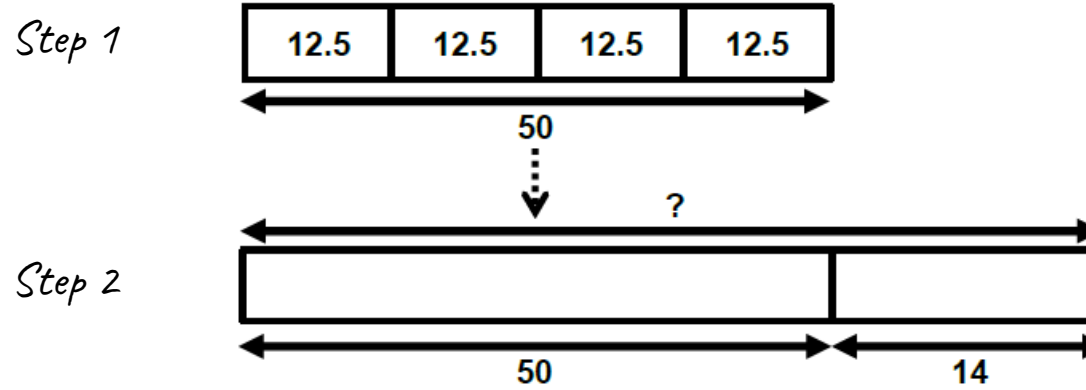
$$225 \div ? = 7 \text{ r } 1$$



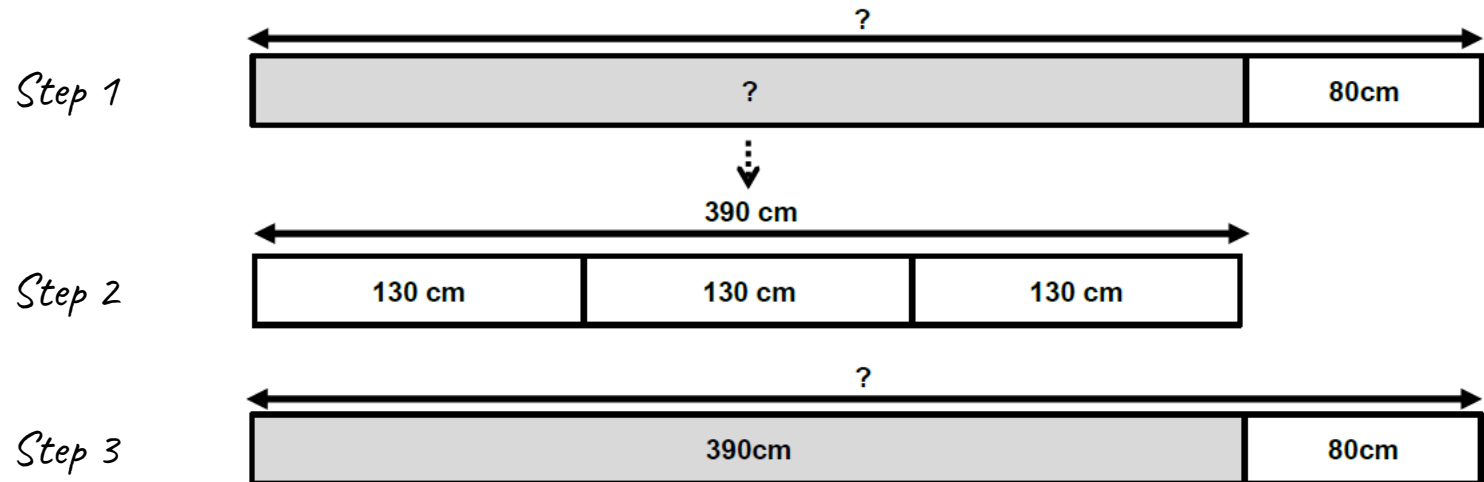
Multiplication and division

- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign

Every day, for 4 days, Sally scored 12.5 on her test. On her fifth day, she scored 14. What was her total score for the week?

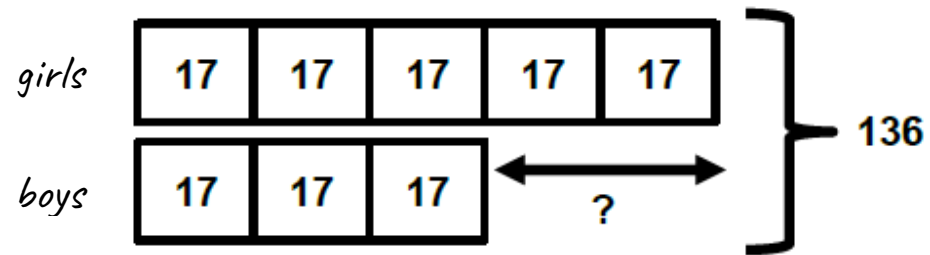
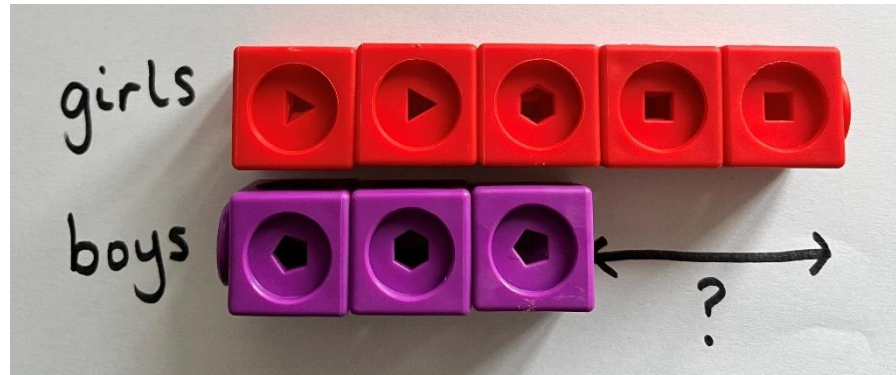


I cut 80cm from a length of ribbon and shared the remainder between 3 friends. Each friend now has 1.3m of ribbon. How much did I start with?



The original model can be completed using the information from step 2.

At the school disco, there are 5 girls to every 3 boys. If there are 136 children at the disco. How many more girls than boys are there?



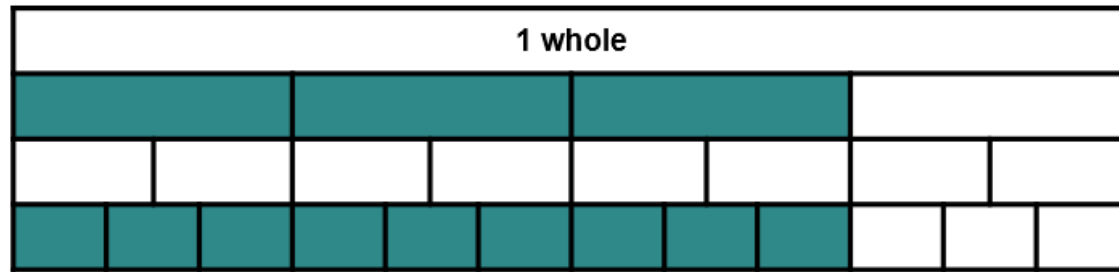
Fractions

- Compare and order fractions whose denominators are all multiples of the same number

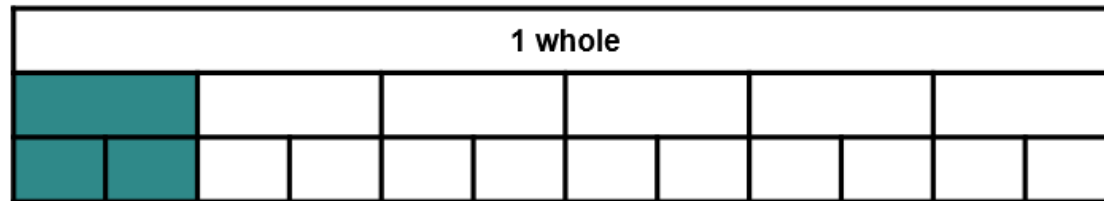
The aim is for children to discover the relationship and the rule for themselves.

Which fraction is greater; $\frac{3}{4}$ or $\frac{1}{6}$?

Three quarters converted into twelfths



One sixth converted into twelfths



Fractions that have the same numerator can be compared using their denominators.

- Which is bigger; $\frac{3}{5}$ or $\frac{3}{6}$?

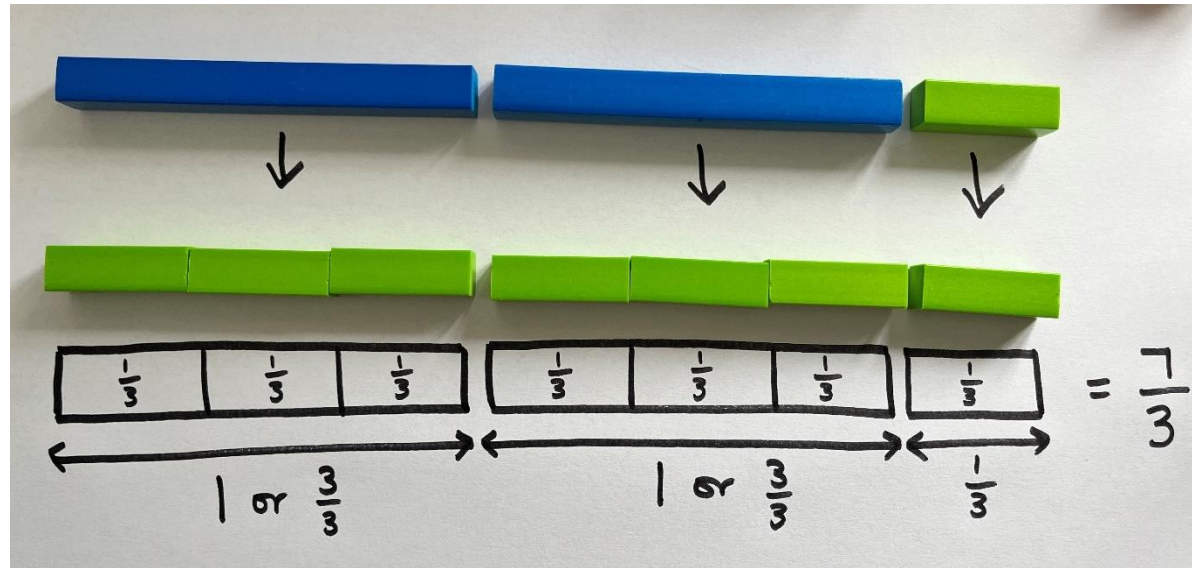
Fractions that have the same denominator can be compared using their numerators.

- Which is bigger; $\frac{2}{10}$ or $\frac{4}{10}$?

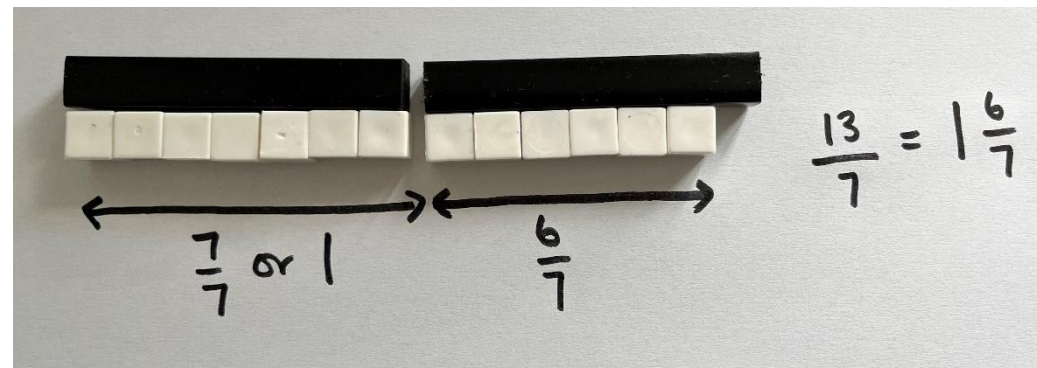
Fractions

- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements >1 as a mixed number

Convert $2\frac{1}{3}$ into an improper fraction.



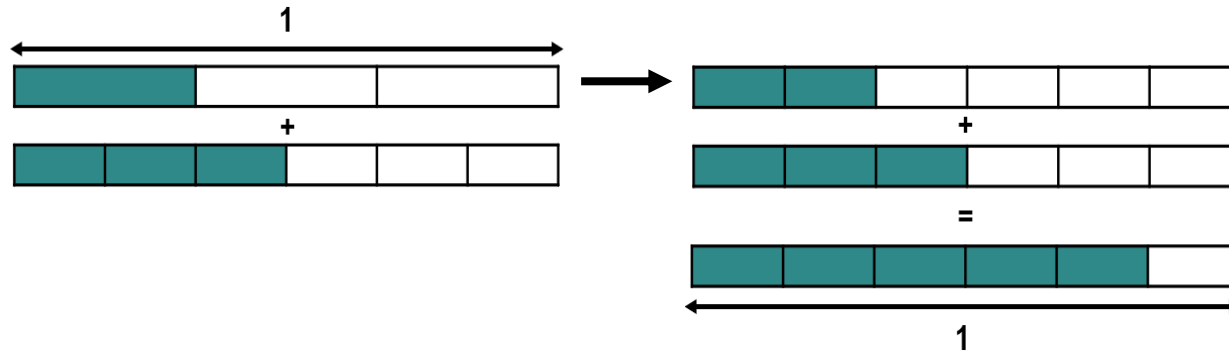
Convert $\frac{13}{7}$ to a mixed number.



Fractions

- Add and subtract fractions with the same denominator and multiples of the same number

$$\frac{1}{3} + \frac{3}{6}$$

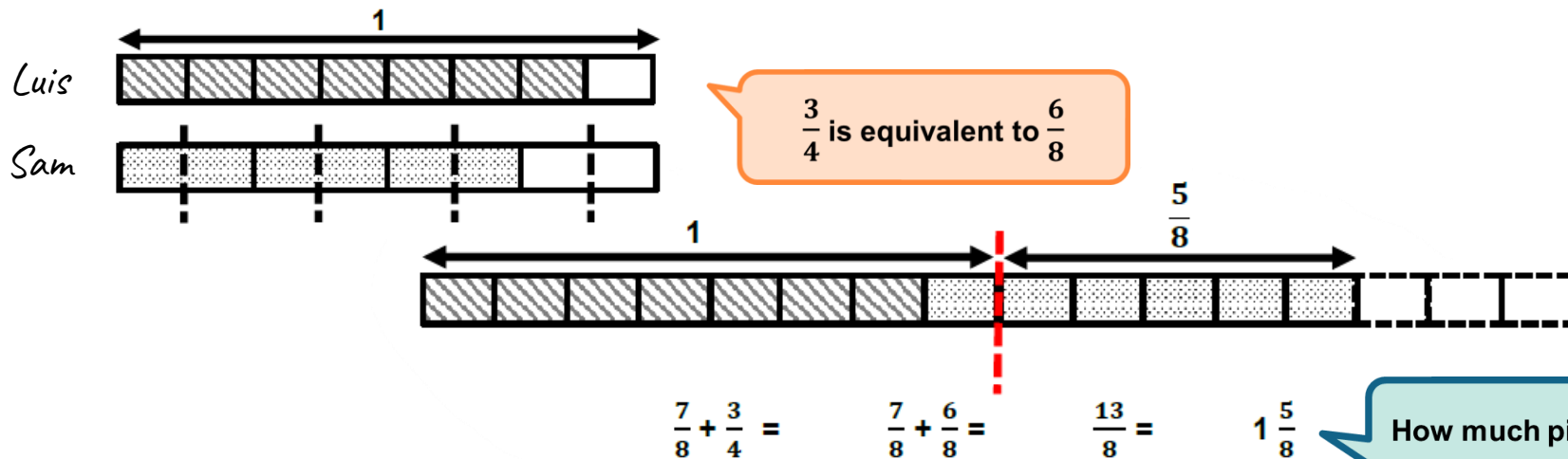


$\frac{1}{3}$ is equivalent to $\frac{2}{6}$

$\frac{3}{6}$ added to $\frac{2}{6}$ equals $\frac{5}{6}$

YEAR 5

Sam and Luis have a pizza each. Sam eats $\frac{3}{4}$ of his pizza. Luis eats $\frac{7}{8}$ of his pizza. How much pizza have they eaten altogether?



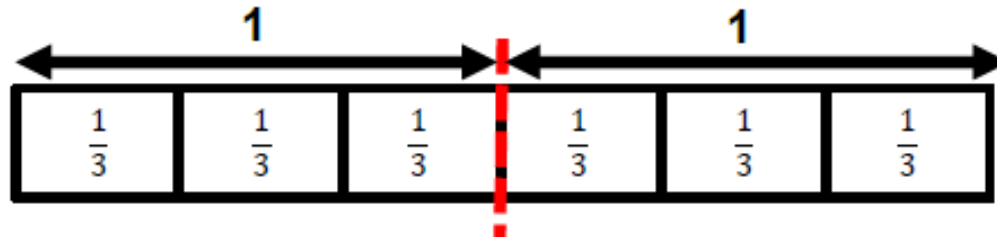
$\frac{3}{4}$ is equivalent to $\frac{6}{8}$

How much pizza is left?

Fractions

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

$$6 \times \frac{1}{3} = ?$$



$$= \frac{6}{3} = 2$$

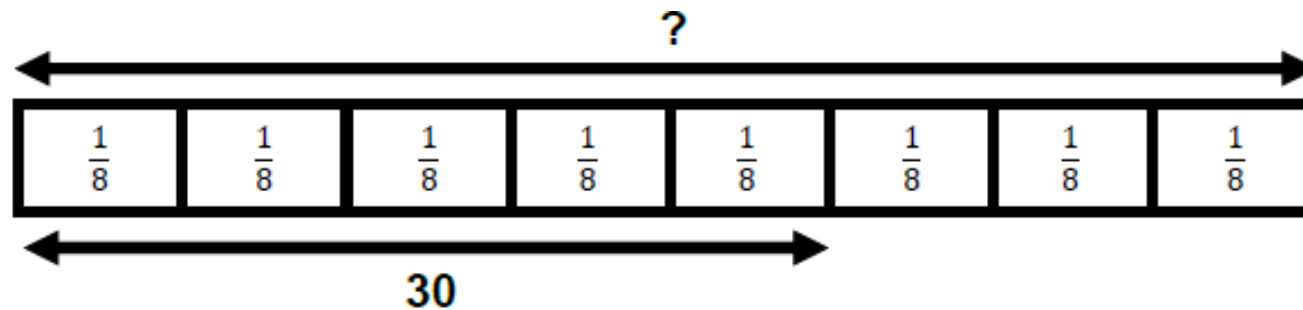
How could you say this?

One third multiplied by six is equal to six thirds.

size of each group x number of groups = total number

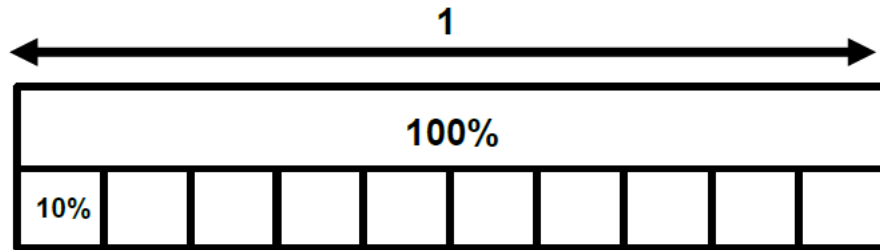
- Solving problems involving fractions

30 is $\frac{5}{8}$ of a number. What is the number?



Fractions (including decimals and percentages)

- Recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator hundred, and as a decimal



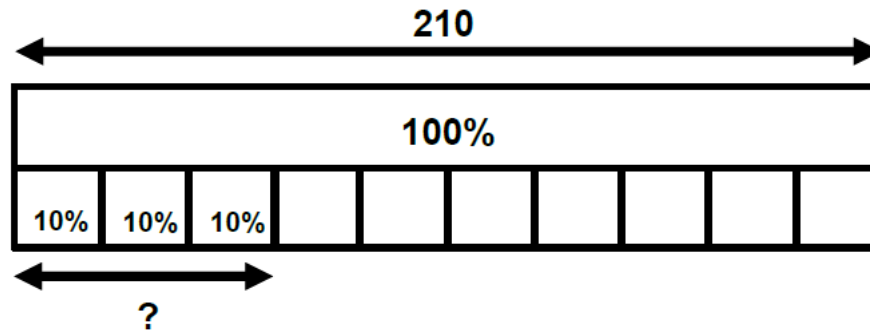
$$10\% = \frac{10}{100} = \frac{1}{10} = 0.1$$

If the whole is 100%, what does each block represent?

If we wanted each block to represent 20%, what would the model look like?

- Solve problems which require knowledge of percentage

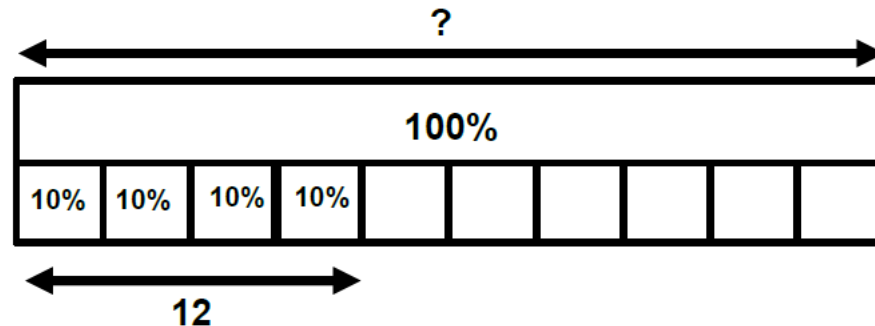
What is 30% of 210?



If the whole is 100%, what does each block represent?

What is the value of each block?

Twelve is 40% of a number. What is the number?



YEAR 6

Fractions (including decimals and percentages)

- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

To add these fractions, children need to convert them into equivalent fractions. Using Cuisenaire rods can support with refreshing their knowledge of equivalent fractions.

$$\frac{1}{2} + \frac{1}{4} = ?$$



$$\frac{1}{2} + \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

The purple rod has been chosen to allow one half and one quarter to be represented on the same model.

If purple is the whole, which colour shows us half?

We can rename half as two quarters.

If purple is the whole, which colour shows us quarters?

Deepen understanding by asking the children if there are other colour rods which would allow us to show halves and quarters on the same model.



$$\frac{1}{2} + \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

Which colour rods will allow us to represent halves and thirds on the same model?

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Fractions (including decimals and percentages)

- Multiply simple pairs of proper fractions, writing the answer in its simplest form

$$1 \times \frac{1}{4} =$$



Why have I selected the purple rod?
What value does it represent?



The value of the purple rod is 1.
We can use a white rod to show $\frac{1}{4}$.

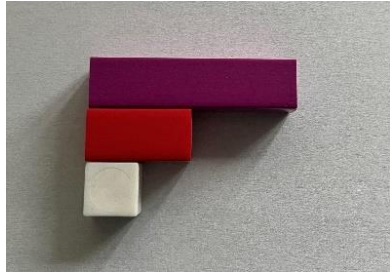
$$3 \times \frac{1}{4} =$$



I have 3 lots of one quarter.

I have three quarters ($\frac{3}{4}$).

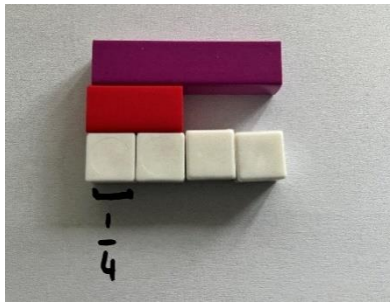
$$\frac{1}{2} \times \frac{1}{2} =$$



If purple is the whole, what is half?

Red shows a half.
White is equal to half of a half.

To find the value of white, children need to refer to the whole.



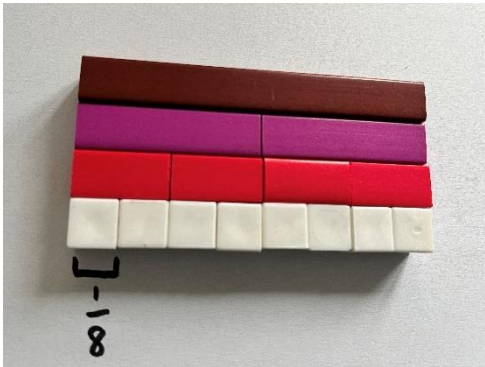
White is one quarter ($\frac{1}{4}$).

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\frac{1}{4} \times \frac{1}{2} =$$

In this case, we need to discover how big one quarter of one half is.

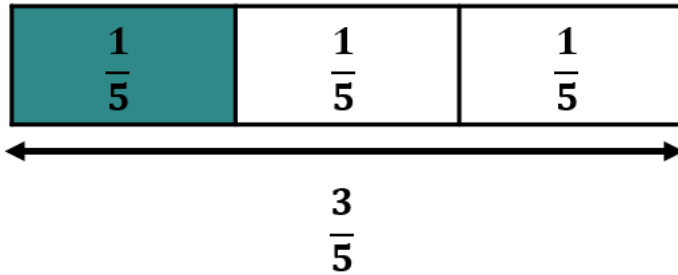
Commutative law could be considered to identify one half of one quarter as equal to one quarter of one half.



$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

- Divide proper fractions by whole numbers

$$\frac{3}{5} \div 3 =$$



If you share ... equally into ... groups, there is ... in each group.

If you share three fifths equally into 3 groups, there is one fifth in each group.

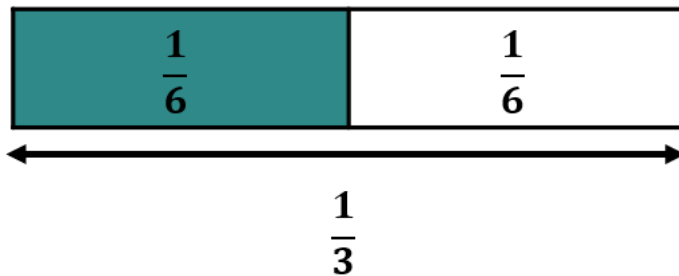
What else can you see?

I can see that $\frac{1}{5}$ is a third of $\frac{3}{5}$.

I can see that $3 \times \frac{1}{5}$ is equal to $\frac{3}{5}$.

I can see how many fifths are in three fifths.

$$\frac{1}{3} \div 2 =$$

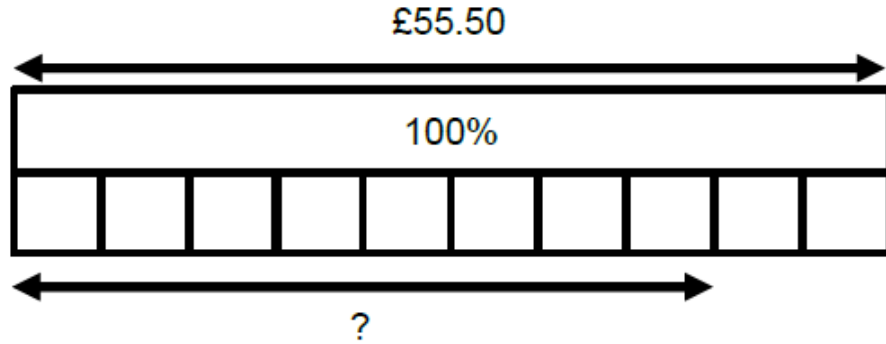


If you share one third equally into 2 groups, there is one sixth in each group.

Fractions (including decimals and percentages)

- Solve problems involving the calculation of percentages

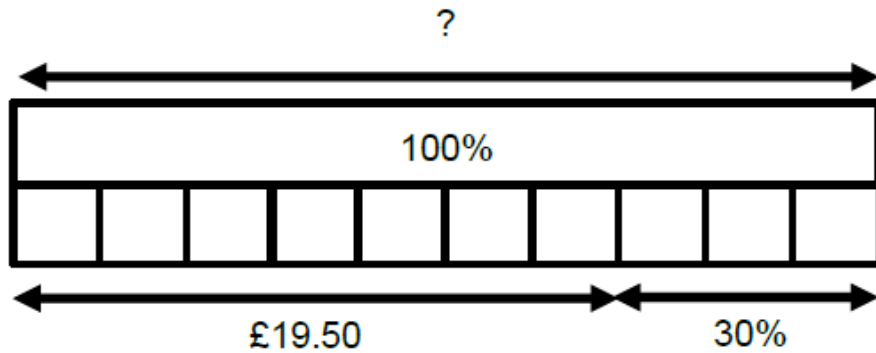
The normal price for a tracksuit is £55.50. How much will it cost in a '20% off' sale?



What is the value of 10%?
How can this be represented on the model?

Can you think of two ways to find the answer?

In a '30% off' sale, Carl pays £19.50 for a t-shirt. How much was the t-shirt before the sale?

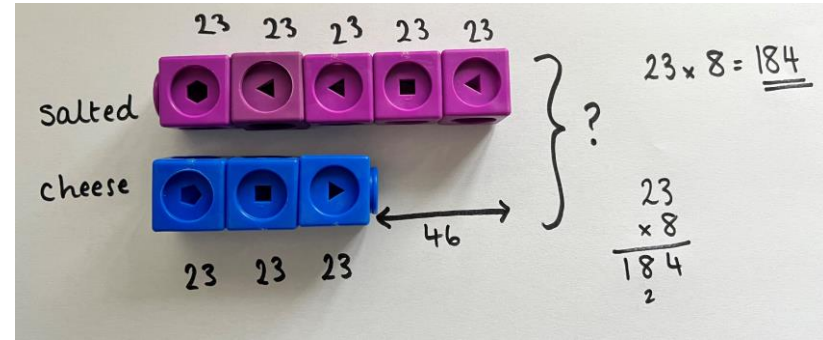
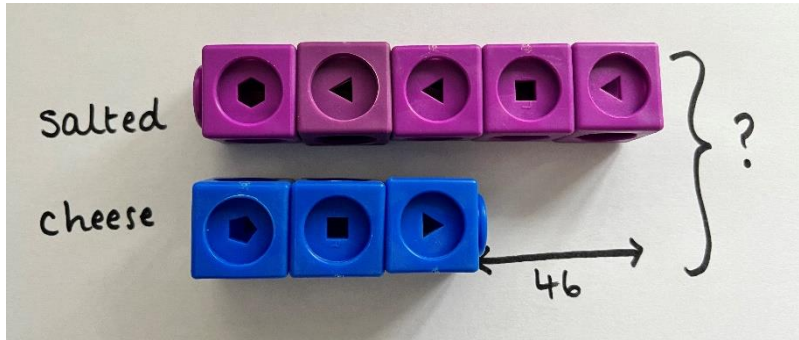


How much has he saved?

Ratio and proportion

- Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

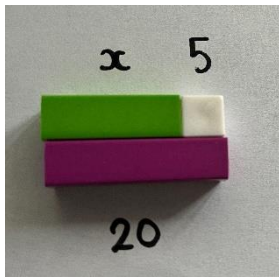
In a survey, the ratio of the number of people who preferred 'ready-salted' to 'cheese and onion' crisps was 5:3. Forty-six more people preferred ready-salted. How many people took part in the survey?



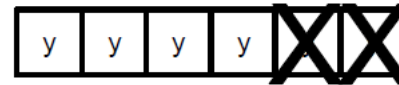
Algebra

- Express missing number problems algebraically

$x + 5 = 20$



$6y - 2y = 4y$



What is $2n + 9$ when $n = 22$?

